Stress Approximation for Elasticity Computations by First-Order System Finite Element Methods

Gerhard Starke[1] Benjamin Müller[2]

In this talk, different finite element methods are examined regarding their ability to produce accurate approximations of the stresses associated with elasticity problems. Such approximations are of interest in many applications in solid mechanics since large local stresses may cause inelastic material behavior or failure. Accurate approximations of surface traction forces also rely on the good resolution of the stress components. The focus of this presentation is on elastic deformations starting from linear elasticity and then turning to hyperelastic models involving geometrical and material nonlinearities. Of particular interest are approaches which remain uniformly accurate in the limit of incompressible materials.

The standard way to perform elasticity computations is based on the representation of the displacement variable by suitable finite element spaces which are augmented, in the incompressible regime, by an additional pressure variable. From these finite element approximations, accurate stresses can be reconstructed in a localizable post-processing step. An alternative approach consists in the use of variational formulations involving the stress as an independent variable which is approximated directly in suitable $H(div)$-conforming finite element spaces. Such approaches may either be of saddle-point or of least-squares type and relations between these two will be investigated in detail.

The approximations obtained from the stress-based finite element approaches will be compared computationally with those obtained from a reconstruction procedure. For all of the above approaches, stress approximations in Raviart-Thomas spaces of lowest and next-to-lowest order will be produced. This is done for some two- and three-dimensional model problems in the linearly elastic as well as the hyperelastic setting including incompressible materials.

References:

[1] B. Müller, G. Starke, A. Schwarz, J. Schröder: A First-Order System Least Squares Method for Hyperelasticity. SIAM J. Sci. Comput. 36: B795-B816 (2014)

[1] Fakultät für Mathematik, Universität Duisburg-Essen, Thea-Leymann-Str. 9, 45127 Essen, gerhard.starke@uni-due.de

[2] Fakultät für Mathematik, Universität Duisburg-Essen, Thea-Leymann-Str. 9, 45127 Essen, benjamin.mueller@uni-due.de