

# Discretization of Control Constrained Optimal Control Problems with Higher Order Finite Elements

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In this talk we consider the optimal control of an elliptic PDE subject to pointwise control constraints. We discretize the problem with higher order finite elements in order to achieve fast convergence with respect to the mesh width  $h$ .

We propose a discretization of the optimal control problem which uses lumping for the mass matrices. This scheme has the advantage that “optimization” and “discretization” commute. We provide a-priori error estimates. In the case of  $P^2$  finite elements, we achieve the order  $h^2$  for the  $L^2$  error in the state and the adjoint state on polygonal convex domains with uniform meshes. Using meshes which are slightly refined at the interface of the active and inactive set, we get convergence of order  $h^3$ , where  $h$  is the mesh-width in the bulk of the domain. We also obtain convergence order  $h^3$  for the optimal control in the nodes of the discretization and for a post-processed control.

The theoretical findings are confirmed by numerical experiments.

This is joint work with Arnd Rösch (Universität Duisburg-Essen).

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