Finite Elements for Mechanochemical Pattern Formation

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In this talk a finite element method for mechanochemical pattern formation will be presented. A biological application of this prototypic model is embryonic development of fertilized cells.

We model biological tissues using the hyperelastic Saint Venant-Kirchhoff model. The growth processes are modeled by splitting the deformation gradient into an active part and an elastic response. The active part depends on the concentration of signaling molecules, which are modeled by an reaction-diffusion equation.

Evolving patterns are reinforced by a feedback mechanism since the experimental observations show that biological cells react to stress and to the change of their shape.
We will present a mechanism using stress as well as a mechanism using strain which is stable under different initial conditions.

Finally, implementation details such as oscillating growth and rotating solutions will be addressed. Large problems, in particular in 3D, are solved with a parallel multi grid solver of the software library Gascoigne 3D.

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