A-priori L^p-Error Analysis for the Obstacle Problem

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This talk is concerned with the accuracy that is obtained when piecewise linear finite elements are used to approximate the solution of the unilateral obstacle problem

$$\min \int_\Omega \nabla v \cdot \nabla v \,\mathrm{d}x - 2\,\langle f, v\rangle$$
$$\text{s.t. } v \in \{w \in H_0^1(\Omega) :\ w \geq \psi \text{ a.e.}\}.$$

The starting point of our investigation is a generalization of Céa's lemma which goes back to Falk and allows to derive a priori estimates for the H^1-error between the exact and the approximate solution. We then move on to some lesser known results by Mosco which can be utilized to extend the classical Nitsche Trick to one-dimensional problems with inequality constraints. After that, it is demonstrated by means of a counter example that a general a priori error estimate of the form

$$\|u - u_h\|_{L^q} \leq C\, h^2$$

for some $1 \leq q \leq \infty$ cannot be obtained for the obstacle problem unless the obstacle ψ is assumed to possess $W^{2,\infty}$-regularity. Using a discrete maximum principle we subsequently derive error estimates in the L^∞-norm which are optimal at least in the one-dimensional case. Lastly, we will discuss some questions which remain open in the multivariate setting.

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