An Assessment of Solvers for Saddle Point Problems Emerging from the Incompressible Navier-Stokes Equations

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This talk presents an assessment of the performance of different solvers for linear saddle point problems. In fluid dynamics large linear saddle point problems emerge from the linearization and discretization of the Navier–Stokes equations. Preconditioned Krylov subspace methods such as FGMRES are a popular choice to solve these large linear systems. We employ FGMRES with two different types of preconditioners: A geometric multigrid preconditioner with different implementations of Vanka type smoothers is compared with the least squares commutator preconditioner (LSC) of Elman et al. The LSC preconditioner has been modified recently as to include boundary conditions and comparisons of the LSC approach with geometric multigrid preconditioners are not available so far. We incorporate the original and the boundary corrected LSC preconditioner into our studies.
For comparison of these preconditioned iterative methods we also consider the direct solvers UMFPACK and PARDISO.
The solvers are appplied to several variants of the commmon benchmark example of a flow around a cylinder, in two and three dimensions as well as for the steady and the time-dependent Navier–Stokes equations.
The methods and examples are implemented in the in-house finite element software package MooN.

References:

[1] H. C. Elman and R. S. Tuminaro, Boundary conditions in approximate commutator preconditioners for the Navier-Stokes equations. ETNA 35: 257–280, 2009.
[2] H. C. Elman, V. E. Howle, J. Shadid, R. Shuttleworth and R. Tuminaro, Block preconditioners based on approximate commutators. SIAM J. Sci. Comput. 27(5): 1651–1668, 2006.
[3] V. John, High order finite element methods and multigrid solvers in a benchmark problem for the 3D Navier-Stokes equations. Int. J. Numer. Meth. Fluids 40: 775–798, 2002.
[4] V. John and G. Matthies, MooNMD - a program package based on mapped finite element methods. Comput Visual Sci. 6: 163–170, 2004.
[5] Y. Saad, A flexible inner-outer preconditioned GMRES algorithm. SIAM J. Sci. Comput. 14(2):461-469, 1993.
[6] M. Schäfer and S. Turek, The benchmark problem “Flow around a cylinder”, in: Flow Simulation with High-Performance Computers II, vol. 52. Notes on Numerical Fluid Mechanics, Vieweg. 1996; 547–566.

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