Discretization of Elliptic Differential Equations Using Sparse Grids and Prewavelets

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Sparse grids can be used to discretize elliptic differential equations of second order on a d-dimensional cube. Using the Galerkin discretization, one obtains a linear equation system with $O(N(logN)^{d-1})$ unknowns. The corresponding discretization error is $O(N^{-1}(logN)^{d-1})$ in the H^1-norm. A major difficulty in using this sparse grid discretization is the complexity of the related stiffness matrix. As a consequence only PDE's with constant coefficients can be efficiently be discretized using the standard sparse grid discretization with $d > 2$. To reduce the complexity of the sparse grid discretization matrix, we apply prewavelets. This simplifies the implementation of the corresponding algorithms. Furthermore, we present a new sparse grid discretization for the discretization of elliptic differential equations with variable coefficients. This discretization utilizes a semi-othogonality property. The convergence rate and stability of the discretization is proven for arbitrary dimensions d.

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