A Modified Pressure-Robust 2nd-Order Finite Element Method for Navier-Stokes Discretisations

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The Taylor-Hood finite element method can be enriched with bubble functions and so allows for piecewise linear pressure (without continuity) in the discretisation of the Navier-Stokes equations. This increases the stability of the method in presence of complicated pressures.

However, there still remains a pressure-dependence of the a priori velocity error estimates due to non divergence-free test functions. This can be remedied by a variational crime in the spirit of the references below that employs local reconstruction of the bubble functions into the Brezzi-Douglas-Marini elements.

Several numerical examples compare and illustrate the robustness of the modified method. In examples with complicated pressures the errors are significantly smaller than for the unstabilised method and much smaller than for the Taylor-Hood finite element method.

References:

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