Space-Time Adaptive Discontinuous Galerkin Methods for Semilinear Diffusion-Convection-Reaction Equations

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An accurate and efficient numerical resolution of interior/boundary layers in convective dominated problems is a challenge over many decades. In nonlinear stationary problems, the nonlinear reaction term produces sharp layers in addition to the spurious oscillations due to the convection. In the non-stationary case, the resolution of such layers is more critical since the nature of the sharp layers may vary as time progresses. In contrast to the stabilized continuous Galerkin finite element methods, discontinuous Galerkin (DG) methods produce stable discretizations without the need for stabilization parameters. Moreover, DG methods are better suited for adaptive strategies.

In this talk, we apply a time-space adaptive algorithm, which utilizes the elliptic reconstruction technique to be able to use the robust (in Péclet number) residual-based a posteriori error estimator, for the convection dominated reactive flow problems. We derive a posteriori error bounds in the $L^2(H^1)$ and $L^\infty(L^2)$-type norms using backward Euler in time and symmetric interior penalty Galerkin (SIPG) in space. We also investigate the influence of the flow field and surface tension on droplet breakup phenomena described by the non-local advective Allen-Cahn equation. In contrast to the reactive flow problems, where the velocity field is incompressible, in the advective Allen-Cahn equation the velocity field is expanding or contracting. After the discretization in time by the implicit Euler method, the resulting sequence of semi-linear elliptic equations are solved by the residual-based adaptive algorithm. Numerical results demonstrating the performance of the adaptive algorithm will be presented for convection dominated problems in reactive flow equations and non-local advective Allen-Cahn equation.

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