Multi-Index Monte Carlo: When Sparsity Meets Sampling

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I will present our Multi Index Monte Carlo (MIMC) method. We propose and analyze a novel Multi-Index Monte Carlo (MIMC) method for weak approximation of stochastic models that are described in terms of differential equations either driven by random measures or with random coefficients. The MIMC method is both a stochastic version of the combination technique introduced by Zenger, Griebel and collaborators and an extension of the Multilevel Monte Carlo (MLMC) method first described by Heinrich and Giles. Inspired by Giles's seminal work, instead of using first-order differences as in MLMC, we use in MIMC high-order mixed differences to reduce the variance of the hierarchical differences dramatically. This in turn gives a new improved complexity result that increases the domain of the problem parameters for which the method achieves the optimal convergence rate, $\mathcal{O}(\mathrm{TOL}^{-2})$. Using optimal index sets that we determined, MIMC achieves a better rate for the computational complexity does not depend on the dimensionality of the underlying problem, up to logarithmic factors. We present numerical results related to a three dimensional PDE with random coefficients to substantiate some of the derived computational complexity rates. Finally, using the Lindeberg-Feller theorem, we also show the asymptotic normality of the statistical error in the MIMC estimator and justify in this way our error estimate that allows prescribing both the required accuracy and confidence in the final result.

References:

[1] Abdul-Lateef Haji-Ali, Fabio Nobile, and Raul Tempone. "Multi-Index Monte Carlo: When Sparsity Meets Sampling.", arXiv preprint arXiv:1405.3757 (2014)

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