On the Numerical Analysis of the Oseen Equations

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Let us consider the stationary Oseen-equations

$$\begin{aligned} -\varepsilon\Delta\mathbf{v} + (\mathbf{b}\cdot\nabla)\mathbf{v} + c\mathbf{v} + \nabla p = f, &\quad \text{in } \Omega \subset \mathbb{R}^2 \\ \operatorname{div} \mathbf{v} = 0, &\quad \text{in } \Omega \\ \mathbf{v} = 0, &\quad \text{on } \partial\Omega \end{aligned}$$

with $0 < \varepsilon \ll 1$ and a vector field $\mathbf{b}$.

Here the components of the velocity $\mathbf{v}$ are coupled via the additional pressure term and the divergence condition. These equations are simplifications of the non-linear Navier-Stokes system and can occur when discretising them.

While the structure of solutions of singularly perturbed convection-diffusion problems is well understood and the kind of layers is known, there is still not enough information known in the case of systems like the Oseen equations. Similarly to the convection-diffusion problems, they contain a small parameter that determines strongly the behaviour of its solution near the boundaries.

In this talk we will give some information on the layer structure of the solution $\mathbf{v}$ near the boundaries. We will make use of the stream-function formulation of the Oseen-equations that transform the problem for $\mathbf{v}$ and p into a fourth-order scalar PDE for ψ where $\mathbf{v} = (\psi_y, -\psi_x)$.

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