Continuous Galerkin-Petrov Methods in Time Combined with Stabilised Finite Element Methods in Space Applied to Time-Dependent Convection-Diffusion Problems

Gunar Matthies[1] Friedhelm Schieweck[2]

We consider the numerical solution of time-dependent convection-diffusion problems by combining continuous Galerkin-Petrov methods (cGP) in time with stabilised finite element methods in space. We will concentrate on symmetric spatial stabilisations like continuous interior penalty methods (CIP) or the local projection stabilisation (LPS).

As main result, error estimates for the fully discrete solution will be given in different norms. One main ingredient of the proofs is a lifting operator which maps the continuous solution trajectory of cGP into a trajectory which is continuously differentiable. This lifting is obtained on each sub interval in time by a post-processing of the cGP solution.

We present error estimates for the post-processed solution of order $\mathcal{O}(\tau^{k+2} + h^{r+1/2})$ for the $L^\infty(L^2)$-norm and the $L^2(L^2)$. Here, h is the spatial discretisation parameter, τ the temporal one, k the polynomial order in time, and r the ansatz order in space.

The theoretical predictions will be confirmed by numerical experiments.

[1] Universität Kassel, FB 10, Institut für Mathematik, Kassel, Germany, matthies@mathematik.uni-kassel.de

[2] Otto-von-Guericke-Universität Magdeburg, Institut für Analysis und Numerik, schiewec@ovgu.de