

# Model Order Reduction for an Acoustic Eigenvalue Problem

Antti Hannukainen<sup>1</sup> Jarmo Malinen<sup>2</sup> Antti Ojalampi<sup>3</sup>

We study a quadratic eigenvalue problem related to the modeling of human speech production: Find  $(\lambda, u) \in (\mathbb{C}, V)$  such that

$$c^2(\nabla u, \nabla v)_{L^2(\Omega)} + \lambda c(u, v)_{L^2(\Gamma)} + \lambda^2(u, v)_{L^2(\Omega)} = 0,$$

in which  $\Omega \subset \mathbb{R}^3$  is the computational domain,  $\Gamma \subset \partial\Omega$  and  $c$  is the speed of sound. The finite element method is used for the spatial discretization, so that the finite-dimensional space  $V$  is a subset of  $H^1(\Omega)$  with appropriate boundary conditions built in. For more details on this model, see [1].

Our interest is in the formants, that is, in the resonant frequencies of the vocal tract. We are especially interested in the effect of surgery or anatomical abnormalities to these resonances. As surgery changes the geometry of the vocal tract, assessing the effect requires solving several eigenvalue problems in different domains  $\Omega$ .

Vocal tract resonances and corresponding geometries were simultaneously measured in [2] using an MRI machine and a special microphone setup. The authors performed an eigenvalue analysis using the geometries extracted from the MRI measurements and a Dirichlet boundary condition placed at the mouth. However, when compared to the measured formants, the computed ones exhibit a large error for certain cases. Based on our numerical experiments, this error is due to the truncation of the computational domain with a Dirichlet boundary condition at the mouth. That is, the effect of the exterior space is not taken into account.

Taking the exterior space into account increases the size of the system considerably, which is not desired. Since the geometry of the MRI-coil remains unchanged throughout the computations, we can reduce the size of the system by using model order reduction techniques in the exterior part of the domain. In this talk, we describe a technique for reducing the size of the system via computing a special basis for the exterior domain. We study the effect of this approach on the accuracy of the computed eigenvalues both numerically and analytically.

## References:

- [1] A. Hannukainen, T. Lukkari, J. Malinen, and P. Palo: Vowel formants from the wave equation. *JASA-EL*, 122(1), 2007, EL1-EL7.
- [2] D. Aalto, O. Aaltonen, R.-P. Happonen, P. Jääsaari, A. Kivelä, J. Kuortti, J.-M. Luukinen, J. Malinen, T. Murtola, R. Parkkola, J. Saunavaara, and M. Vainio: Large scale data acquisition of simultaneous MRI and speech, *Appl. Acoust.*, 83(1), 2014, 64-75.

---

<sup>1</sup> Aalto University School of Science, Department of Mathematics and Systems Analysis, Aalto, Finland,  
[antti.hannukainen@aalto.fi](mailto:antti.hannukainen@aalto.fi)

<sup>2</sup> Aalto University School of Science, Department of Mathematics and Systems Analysis,  
[jarmo.malinen@aalto.fi](mailto:jarmo.malinen@aalto.fi)

<sup>3</sup> Aalto University School of Science, Department of Mathematics and Systems Analysis,  
[antti.ojalampi@aalto.fi](mailto:antti.ojalampi@aalto.fi)