Anisotropic mesh refinement in polyhedral domains: error estimates with data in $L^2(\Omega)$

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The presentation is concerned with the finite element solution of the Poisson equation with homogeneous Dirichlet boundary condition in a three-dimensional domain. Anisotropic, graded meshes [2] are used for dealing with the singular behavior of the solution in the vicinity of the non-smooth parts of the boundary. The discretization error is analyzed for the piecewise linear approximation in the $H^1(\Omega)$- and $L^2(\Omega)$-norms by using a new quasi-interpolation operator. This new interpolant is introduced in order to prove the estimates for $L^2(\Omega)$-data in the differential equation which is not possible for the standard nodal interpolant. These new estimates allow for the extension of certain error estimates for optimal control problems with elliptic partial differential equation and for a simpler proof of the discrete compactness property for edge elements of any order on this kind of finite element meshes, see [1].

References:

[1] Th. Apel, A. L. Lombardi, and M. Winkler. Anisotropic mesh refinement in polyhedral domains: error estimates with data in $L^2(\Omega)$. Preprint arXiv:1303.2960 [math.NA], http://arxiv.org/abs/1303.2960, 2013

[2] Th. Apel and S. Nicaise. The finite element method with anisotropic mesh grading for elliptic problems in domains with corners and edges. *Math. Methods Appl. Sci.*, 21:519–549, 1998.

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