Arbitrary order BEM-based FEM on star-shaped elements

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In the development of numerical methods to solve boundary value problems the requirement of flexible mesh handling gains more and more importance. The BEM-based finite element method is one of the new promising strategies which yields conforming approximations on polygonal and polyhedral meshes, respectively. This flexibility is obtained by special trial functions which are defined implicitly as solutions of local boundary value problems related to the underlying differential equation. These functions are treated by means of boundary element methods (BEM) in the realization.

The first part of the presentation gives a short introduction into the BEM-based FEM and deals with recent developments. Here, the definitions of lower order trial functions are discussed for two and three space dimensions. Furthermore, it is shown that the method can be applied to mixed FEM formulations involving $H(\mathrm{div})$-conforming approximations on polygonal meshes. In the second part, ideas from the previous work [SIAM J. Numer. Anal., 50(5):2357–2378, 2012] are generalized to construct trial functions which yield arbitrary order of convergence. With the help of an appropriate interpolation operator it is possible to prove convergence rates in the H^1- as well as in the L_2-norm for the BEM-based FEM on polygonal meshes with star-shaped elements. Several numerical experiments confirm the theoretical results.

References:

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