Higher order space-time discretizations for hyperbolic and parabolic problems I: Motivation and schemes

Markus Bause[1] Uwe Köcher

In the field of numerical approximation of time dependent parabolic differential equations higher order variational time discretizations have recently attracted the interest of researchers. By using a Galerkin approach we have a uniform variational approach in space and time which is advantageous for the analysis of the fully discrete system as well for the construction of simultaneous space-time adaptive methods or residual-based stabilization schemes in the case of convection-dominated transport problems. Further, it is very natural to construct methods of higher order and well-known finite element concepts can be used to obtain at least A-stable discretizations.

In this contribution we present continuous and discontinuous Galerkin time discretizations for second order hyperbolic problems and a class of parabolic equations. Our interest in developing numerical approximation schemes of higher order accuracy for the hyperbolic wave equation comes from mechanical engineering. Material inspection by piezoelectric induced ultrasonic waves is a relatively new and an intelligent technique to monitor the health of light-weight structures (e.g. carbon fiber reinforced plastics), for damage detection (delamination, matrix cracks, fiber breaks) and non-destructive evaluation. Here, high accuracy of the numerical calculations is of particular importance. As far as parabolic problems are concerned, we are interested in studying coupled systems of convection-diffusion-reaction equations as they typically arise in mechanical and environmental engineering (e.g. flow and transport in the subsurface). Here, variational techniques might help in the future to develop new space-time stabilization techniques in the case of convection-dominated transport and to apply multiscale and upscaling techniques that are known from the spatial discretization to the time variable.

In this contribution we introduce the underlying mathematical models and present their space-time Galerkin discretizations. For the spatial discretization of the wave equation the interior penalty discontinuous Galerkin method is used. Mixed finite element approaches are applied to discretize the spatial variables of the parabolic problems. Qualitative properties of the discretization schemes are discussed.

[1] Helmut Schmidt University, University of the Federal Armed Forces Hamburg, Faculty of Mechanical Engineering, Hamburg, Germany,
bause@hsu-hh.de