Abstract L^2 analysis for variational inequalities of Signorini type

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This talk is concerned with variationally consistent Lagrange multiplier based discretizations for Signorini type problems. We start with L^2 a priori estimates for well known low order finite element discretizations and then discuss modern discretizations within the framework of isogeometrical analysis.

While there is a series of results on the convergence rate in the H^1-norm for the primal solution and the $H^{-1/2}$-norm for the dual solution, there is hardly any result on the L^2-norm for the different solution components. Here we give new quasi-optimal results for low order discretizations based on the use of the Aubin–Nitsche trick. We point out that due to the inequality character of the problem the dual problem has reduced regularity, and thus standard techniques only yield sub-optimal results.

Motivated by these a priori results, we then focus on formulations based on the isogeometric approach. Here we present two alternatives: One is based on the use of a set of biorthogonal Lagrange multipliers, whereas the second one works with a trace space of possibly different order. Here we discuss advantages and disadvantages of these two strategies and provide abstract criteria for optimality.

Numerical examples illustrate the theoretical results and show quantitative and qualitative effects of the influence

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