# Finite Element simulations for a coupled bulk-surface convection diffusion problem

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We consider a convection diffusion equation in a bulk $\Omega \in \mathbb{R}^3$ coupled to a diffusion equation on its closed boundary $\Gamma$

$$\partial_t c - D\Delta c + (u \cdot \nabla)c = f \qquad \text{in } \Omega \tag{1}$$

$$D\partial_n c = -S(c, c_\Gamma) \qquad \text{on } \Gamma \tag{2}$$

$$\partial_t c_\Gamma - D_\Gamma \Delta_\Gamma c_\Gamma = g + S(c, c_\Gamma) \qquad \text{on } \Gamma. \tag{3}$$

The velocity field $u$ is given and fulfills the pointwise constrains $\nabla \cdot u = 0$ and $u|_\Gamma = 0$. A linear and a nonlinear version of the source term are regarded

$$S(c, c_\Gamma) = k_a c - k_d c_\Gamma, \quad S(c, c_\Gamma) = k_a c(1 - c_\Gamma) - k_d c_\Gamma \quad (k_a, k_d > 0).$$

In this talk we present a higher order finite element discretization of the above problem on polyhedral domains. The bulk mesh consists of tetrahedrals and the surface mesh is its restriction to the surface and therefore triangular. For the basis functions on the surface we use a restriction of the bulk basis functions. Time discretization is made by a Crank-Nicolson scheme.

The equations are decoupled by an approximation of the source term based on former solutions for $c$ and $c_\Gamma$, respectively. This leads to an iterative scheme in every timestep. In the $n$-th timestep for given $c_n^k$, $k \geq 0$, with $c_n^0 = c(t_{n-1})$ the surface equation (3) is solved as a diffusion reaction equation to get $c_{\Gamma,n}^k$. Using $c_{\Gamma,n}^k$ in equations (1) and (2) we get a Robin type convection diffusion problem. Its solution is a new approximation $c_n^{k+1}$ of $c(t_n)$.

Using this decoupled algorithm we perform time dependent numerical simulations. We compare their steady states for $t \to \infty$ to the results of the associated stationary problems.

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