Mixed LSFEM for geometrically and physically nonlinear elasticity problems

Benjamin Müller[1] Gerhard Starke[2] Jörg Schröder[3] Alexander Schwarz[4] Karl Steeger[5]

Deformation processes of solid materials are omnipresent and can be described by systems of partial differential equations in continuum mechanics. In this talk we present a least squares finite element method based on the momentum balance and the constitutive equation for hyperelastic materials. Our approach is motivated by a well-studied least squares formulation for linear elasticity. This method is generalized to an approach which takes physical as well as geometrical nonlinearities into account. The novelty of our approach is that, in addition to the displacement $\mathbf{u}$, we consider the full first Piola-Kirchhoff stress tensor $\mathbf{P}$ and approximate both simultaneously.

In the discrete formulation we use quadratic Raviart-Thomas elements for the stress tensor and continuous piecewise quadratic elements for the displacement vector. For the minimization of the nonlinear least squares functional, the Gauss-Newton method with backtracking line search is used.

A further important aspect in praxis besides the quantification of the discretization error is the evaluation of the model error. We will present a possibility of our approach to decide whether we need only a simple model (e.g. a linear model) on a particular element of our triangulation or we need a more complex model (e.g. a nonlinear model) on it.

At the end of the talk we will illustrate the performance of our method for some two dimensional problems in plane strain configuration and some three dimensional problems.

References:

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[1] Universität Duisburg-Essen, Fakultät für Mathematik, Essen, Germany, benjamin.mueller@uni-due.de

[2] Universität Duisburg-Essen, gerhard.starke@uni-due.de

[3] Universität Duisburg-Essen, j.schroeder@uni-due.de

[4] Universität Duisburg-Essen, alexander.schwarz@uni-due.de

[5] Universität Duisburg-Essen, karl.steeger@uni-due.de