FAKULTÄT FÜR
MATHEMATIK

Chemnitz FEM-Symposium 2013

Programme

Collection of abstracts

List of participants

Chemnitz
FEM
Symposium

TECHNISCHE UNIVERSITÄT
CHEMNITZ

Chemnitz, September 23 - 25, 2013

TECHNISCHE UNIVERSITÄT CHEMNITZ

Fakultät für Mathematik

Chemnitz FEM-Symposium 2013

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Scientific topics:

The symposium is devoted to all aspects of finite elements and wavelet methods in partial differential equations.

The topics include (but are not limited to)

- adaptive methods,
- parallel implementation,
- high order methods.

This year we particularly encourage talks on

- Multiscale Methods
- Isogeometric Analysis
- Spacetime DG

Invited Speakers:

Robert Scheichl (University of Bath)

Bert Jüttler (Johannes Kepler Universität Linz)

Jens Lang (TU Darmstadt)

Scientific Committee:

Th. Apel (München), S. Beuchler (Bonn), G. Haase (Graz),
H. Harbrecht (Basel), R. Herzog (Chemnitz), M. Jung (Dresden),
U. Langer (Linz), A. Meyer (Chemnitz), A. Rösch (Duisburg),
O. Steinbach (Graz)

Organising Committee:

H. Schmidt, S. Mach, I. Riedel, R. Schneider, M. Pester, K. Seidel, A.-K. Glanzberg

http://www.tu-chemnitz.de/mathematik/fem-symposium/

Programme

Programme for Monday, September 23, 2013

9:00 Opening
Room: "Terrassensaal"

Multiscale Methods
Chairman: A. Meyer
Room: "Terrassensaal"

9:10 **Robert Scheichl** 10
Novel Monte Carlo FE methods for random elliptic PDEs.

10:00 **Matthias Maier** 11
A posteriori error estimation for a heterogeneous multiscale finite element method.

10:25 *Tea and coffee break*

Fluid Dynamics
Chairman: S. Franz
Room: "Terrassensaal"

10:50 **Dirk Broersen** 12
A Petrov-Galerkin discretization with optimal test space of a mild-weak formulation of convection-diffusion equations in mixed form.

11:15 **Katharina Höhne** 13
On the asymptotic analysis of the stationary Oseen equations.

11:40 **Gunar Matthies** 14
Higher order variational time discretisations for the Oseen equations.

12:05 **Matthias Gsell** 15
Direct numerical simulation of incompressible fluids.

Contact Problems / Elasticity
Chairman: A. Meyer
Room: "Annaberg-Buchholz"

10:50 **Martina Weise** 16
On the $\mathcal{Q}_2$-$\mathcal{Q}_0$-element for (nearly) incompressible material under large deformations.

11:15 **Benjamin Müller** 17
Mixed LSFEM for geometrically and physically nonlinear elasticity problems.

11:40 **Elena Rukavishnikova** 18
The finite element method for Dirichlet problem with strong singularity of solution on the boundary.

12:05 **Viktor Rukavishnikov** 19
Weighted finite element method for the elasticity problem with singularity.

12:30 *Lunch*

Time-Dependent Problems
Chairman: T. Apel
Room: "Terrassensaal"

14:00	**Jens Lang** .... 20 Convergence and adaptivity at the PDE/stiff ODE interface.
14:50	**Markus Bause** .... 21 Higher order space-time discretizations for hyperbolic and parabolic problems I: Motivation and schemes.
15:15	**Uwe Köcher** .... 22 Higher order space-time discretizations for hyperbolic and parabolic problems II: Algebraic solver and numerical studies.
15:40	*Tea and coffee break*

	Time-Dependent Problems *Chairman*: J. Lang *Room*: "Terrassensaal"	**Fluid Dynamics / Singularly Perturbed Problems** *Chairman*: R. Scheichl *Room*: "Annaberg-Buchholz"
16:15	**Lennard Kamenski** .... 23 Stability of explicit time integration schemes for finite element approximations of linear parabolic equations with anisotropic meshes..	**Kristin Held** .... 26 Finite Element simulations for a coupled bulk-surface convection diffusion problem.
16:40	**Stephan Weller** .... 24 Higher order Galerkin methods as time discretization for free surface flows.	**Sebastian Franz** .... 27 Superconvergence for higher-order Galerkin FEM for convection-diffusion problems.
17:05	**Karen Kuhn** .... 25 Stability and efficiency of a recursive multirate Rosenbrock method.	**Miloslav Vlasak** .... 28 A priori error analysis of nonlinear singularly perturbed problems.
17:30		**Alexander Linke** .... 29 On the role of the Helmholtz decomposition in mixed methods for incompressible flows and a new variational crime.
19:00	*Conference Dinner*	

Programme for Tuesday, September 24, 2013

Isogeometric Analysis
Chairman: G. Haase
Room: "Terrassensaal"

9:00 **Bert Jüttler** 30
Adaptive refinement in isogeometric analysis.

9:50 **Sebastian Westerheide** 31
A geometry-independent framework for coupled volume and surface problems on evolving domains.

10:15 *Tea and coffee break*

	Fluid Dynamics / Singularly Perturbed Problems *Chairman*: M. Bause *Room*: "Terrassensaal"	**Variational Inequalities** *Chairman*: O. Steinbach *Room*: "Annaberg-Buchholz"
10:45	**Fleurianne Bertrand** .... 32 Least squares methods with interface approximation for two phase Stokes flow.	**Steffen Weißer** .... 36 Arbitrary order BEM-based FEM on star-shaped elements.
11:10	**Giulia Deolmi** .... 33 Effective boundary conditions for compressible flows over a rough surface.	**Linus Wunderlich** .... 37 Abstract L^2 analysis for variational inequalities of Signorini type.
11:35	**Khalid Adrigal** .... 34 Higher order variational time discretisations for convection-diffusion problems in time-dependent domains.	**Peter Gangl** .... 38 Topology optimization of electric machines based on topological sensitivity analysis.
12:00	**Steffen Münzenmaier** .... 35 Least-squares finite element methods for coupled generalized Newtonian Stokes-Darcy flow.	**Jörg Frohne** .... 39 Efficient numerical methods for the large-scale parallel solution of thermomechanical contact problems under consideration of mesh separation.

12:30 *Conference Photo*

Lunch

14:30 *Excursion*

Programme for Wednesday, September 25, 2013

	Software *Chairman*: R. Schneider *Room*: "Terrassensaal"
9:00	**Gundolf Haase** . . . . 40 Non-linear FEM solvers on accelerator devices.
9:25	**Lars Ludwig** . . . . 41 SOFE - a new simple and sophisticated object oriented hp-FEM software in MATLAB.
9:50	**Alexander Nikolas Breuer** . . . . 42 Towards optimal performance of SeisSol, an unstructured ADER-DG code..
10:15	**Stefan Findeisen** . . . . 43 First step towards parallel and adaptive computation of Maxwell's equations.
10:40	*Tea and coffee break*
	Anisotropic Problems / A-posteriori Error Estimators *Chairman*: G. Matthies *Room*: "Terrassensaal"
11:10	**Soleiman Yousef** . . . . 44 A posteriori error estimates, stopping criteria, and adaptivity for multiphase compositional flows.
11:35	**Rene Schneider** . . . . 45 Adaptive anisotropic mesh refinement based hierarchical refinement indicators.
12:00	**Ariel Lombardi** . . . . 46 The discrete compactness property for edge elements on anisotropic meshes.
12:25	**Thomas Apel** . . . . 47 Anisotropic mesh refinement in polyhedral domains: error estimates with data in $L^2(\Omega)$.
12:50	Closing
13:00	*Lunch*

Collection of abstracts

Novel Monte Carlo FE methods for random elliptic PDEs

Robert Scheichl[1]

One of the key tasks in many areas of science and engineering is an efficient treatment of data uncertainties and the quantification of how these uncertainties propagate through the system. Subsurface flow is a prime example. The permeability of the subsurface is typically varying over many orders of magnitude, yet our information about the precise values is extremely limited. On the other hand subsurface flow simulations are paramount for safety assessments of longterm radioactive waste repositories or of novel hydrocarbon extraction techniques such as fracking. The mathematical challenges associated with uncertainty quantification are high-dimensional quadrature problems with integrands that involve the solution of PDEs with random coefficients. Due to the heterogeneity of the subsurface and the complexity of the flow, FE simulations of realisations of the integrand are very costly and so it is paramount to make existing uncertainty quantification tools more efficient. Although spectral methods, such as stochastic Galerkin or stochastic collocation type approaches, have recently been shown to be highly effective alternatives, due the curse of dimensionality their applicability is confined to $O(10-100)$ dimensions, which is not sufficient in subsurface flow. The only methods that do not suffer from this curse of dimensionality are Monte Carlo type methods. In this talk I will present new theoretical and numerical results on how to use deterministic Quasi-Monte Carlo sampling rules and hierarchies of finite element models (the so-called multilevel Monte Carlo method) to significantly accelerate the classical Monte Carlo method by a factor of 10-100 on realistic model problems. The analysis reduces to a careful application of classical regularity and finite element approximation error analysis.

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A posteriori error estimation for a heterogeneous multiscale finite element method

Matthias Maier[1]

A big class of modeling problems in physics and engineering is of multiscale character, meaning, that relevant physical processes act on highly different length scales. Due to the huge computational costs that are typically connected with multiscale problems, multiscale schemes were developed to avoid the problem to resolve such problems in full.

Typically, an effective model is solved on a coarse scale and effective parameters are determined with the help of localized sampling problems on a fine scale. However, this introduces significant complexity with respect to sources of error—discretization errors on coarse and fine scale as well as a modeling error for the effective model.

This talk will present a Heterogeneous Multiscale Finite Element scheme for elliptic advection-diffusion problems together with an adaptive framework based on a posteriori error estimation with the help of the Dual Weighted Residual method.

An error splitting will allow for a quantitative error assessment of the different sources of error. Furthermore, a model adaptive approach based on the a posteriori error estimation will be presented.

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A Petrov-Galerkin discretization with optimal test space of a mild-weak formulation of convection-diffusion equations in mixed form

Dirk Broersen[1] Rob Stevenson[2]

Inspired by the Discontinuous Petrov-Galerkin method [*Numer. Methods Partial Differential Equations*, 27 (2011), 70–105] by Demkowicz and Gopalakrishnan, we present a variational formulation of convection-diffusion equations, that is obtained by piecewise integrating one of the two equations in the system w.r.t. a partition of the domain into mesh cells. We apply a Petrov-Galerkin discretization with optimal test functions, or equivalently, minimize the residual in the natural norm associated to the variational form. These optimal test functions can be found by solving local problems.

The available freedom in the method is used to allow a (smooth) passing to a converging method in the convective limit, being a necessary condition to retain convergence and having a bound on the cost for a vanishing diffusion. With several numerical examples, the robust behavior of the method for vanishing diffusion will be illustrated.

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On the asymptotic analysis of the stationary Oseen equations

Katharina Höhne[1] Sebastian Franz[2]

We consider the stationary Oseen equations

$$\begin{aligned} -\varepsilon\Delta u + b\cdot\nabla u + cu + \nabla p &= f \qquad \text{in } \Omega \\ \operatorname{div} u &= 0 \qquad \text{in } \Omega \\ u &= 0 \qquad \text{on } \Gamma \end{aligned}$$

with $0 < \varepsilon \ll 1$. We get these equations by linearization of the Navier-Stokes equations. The solutions of such singularly perturbed differential equations typically exhibit boundary layers. There exists many literature for convection-diffusion type equations. Here, we go a step further. We have a new variable p, the pressure, and there is the special condition for the incompressibility $\operatorname{div} u = 0$. That makes the analysis more complicated.

Our goal is to decompose the solution into a regular part and layer parts. If we know the structure of the boundary layer, we are able to construct a mesh for the FEM, which has better properties than an equidistant mesh. It is well known, that we can reduce oscillations of the numerical solution by layer-adapted meshes.

In this talk, we will present our findings.

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Higher order variational time discretisations for the Oseen equations

Gunar Matthies[1] Naveed Ahmed[2]

We discuss different time discretisations of variational type applied to the Oseen equations. As spatial discretisation, we will consider both inf-sup stable and equal-order pairs of finite element spaces for approximating velocity and pressure.

Since Oseen problems are generally convection-dominated, a spatial stabilization is needed. We will concentrate on local projection stabilization methods which allow to stabilize the streamline derivative, the divergence constraint and, if needed, the pressure gradient separately.

To discretize in time, continuous Galerkin-Petrov methods (cGP) and discontinuous Galerkin methods (dG) as higher order variational time discretisation schemes are applied. These methods are known to be A-stable (cGP) or even strongly A-stable (dG).

Using a simple postprocessing, both velocity and pressure show at the discrete time points a convergence rate of $2k+1$ for dG(k) and $2k$ for cGP(k), respectively.

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Direct numerical simulation of incompressible fluids

Matthias Gsell[1] Olaf Steinbach[2]

We consider the variational formulation of the Navier Stokes equations for incompressible fluids in the stationary and in the instationary case. After summarizing results on unique solvability for both cases, we discuss some linearization strategies and discretization techniques, in particular, an implicit Euler method and a finite element approximation with Taylor Hood elements. Finally we present some numerical examples of pipe flows with large Reynold's numbers in two and three space dimensions.

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On the $\mathcal{Q}_2$-$\mathcal{Q}_0$-element for (nearly) incompressible material under large deformations

Martina Weise[1] Arnd Meyer[2]

In recent years the fast and efficient simulation of modern materials has become more and more important since it is a useful tool to study the behavior of mechanical components under strain without performing an expensive experiment. One of these modern materials is the (nearly) incompressible non-linear elastic material which is characterized by an (almost) infinite bulk modulus. This means that under any load it may change its shape but it will keep its initial volume.

In this talk we will present an adaptive mixed finite element method to determine both the deformation U and the hydrostatic pressure P of a three-dimensional domain Ω in the context of large deformations for (nearly) incompressible material. Starting from the non-linear weak formulation of the deformation problem the mathematical description leads to a saddle point problem. This mixed formulation allows the usage of a mixed FEM. Based on a hexahedral discretisation of the domain Ω we want to apply the $\mathcal{Q}_2$-$\mathcal{Q}_0$ finite element and compare its performance to the Taylor-Hood element in numerical tests.

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Mixed LSFEM for geometrically and physically nonlinear elasticity problems

Benjamin Müller[1] Gerhard Starke[2] Jörg Schröder[3] Alexander Schwarz[4] Karl Steeger[5]

Deformation processes of solid materials are omnipresent and can be described by systems of partial differential equations in continuum mechanics. In this talk we present a least squares finite element method based on the momentum balance and the constitutive equation for hyperelastic materials. Our approach is motivated by a well-studied least squares formulation for linear elasticity. This method is generalized to an approach which takes physical as well as geometrical nonlinearities into account. The novelty of our approach is that, in addition to the displacement $\mathbf{u}$, we consider the full first Piola-Kirchhoff stress tensor $\mathbf{P}$ and approximate both simultaneously.
In the discrete formulation we use quadratic Raviart-Thomas elements for the stress tensor and continuous piecewise quadratic elements for the displacement vector. For the minimization of the nonlinear least squares functional, the Gauss-Newton method with backtracking line search is used.
A further important aspect in praxis besides the quantification of the discretization error is the evaluation of the model error. We will present a possibility of our approach to decide whether we need only a simple model (e.g. a linear model) on a particular element of our triangulation or we need a more complex model (e.g. a nonlinear model) on it.
At the end of the talk we will illustrate the performance of our method for some two dimensional problems in plane strain configuration and some three dimensional problems.

References:

[1] *Z. Cai, G. Starke*: Least-Squares Methods for Linear Elasticity. SIAM J. Numer. Anal. Vol. 42 (2004), 826-842.
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The finite element method for Dirichlet problem with strong singularity of solution on the boundary

Elena Rukavishnikova[1]

We consider the first-boundary-value problem for a non-self-adjoint second-order elliptic equation with coordinated degeneracy of input data whose solution has strong singularity on the curvilinear boundary of a two-dimensional convex domain. For this problem we define the solution as an R_ν-generalized one; we prove its existence and uniqueness in the Sobolev weighted space. We construct and investigate the finite element method for this problem. For that purpose the domain is divided quasi-uniformly into triangles. We introduce a finite element space which contains singular functions whose form depends on the space, to which the R_ν-generalized solution of the problem belongs. It was established that the approximation to the exact-generalized solution has first-order convergence in the norm of the Sobolev weighted space.

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Weighted finite element method for the elasticity problem with singularity

Viktor Rukavishnikov[1]

We consider the two-dimensional elasticity problem with a singularity caused by the presence of a reentrant corner on the domain boundary. For this problem, the notion of the R_ν-generalized solution is introduced and the corresponding definition is used to construct a scheme of the weighted finite-element method (FEM). The proposed method provides a first-order convergence of the approximate solution to the exact one with respect to the mesh step in the $\mathbf{W}^1_{2,\nu}(\Omega)$-norm. The convergence rate does not depend on the size of the angle and kind of the boundary conditions imposed on its sides. This statement is illustrated by the results of numerical experiments. For the model problems, the dependence of the rate of convergence of the approximate solution to the exact R_ν-generalized solution with respect to the regularization parameters δ and ν is investigated.

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Convergence and adaptivity at the PDE/stiff ODE interface

Jens Lang[1]

In this talk I will emphasize on the use of different time integrators in combination with adaptive finite element discretizations in space. After briefly discussing general approaches for combining adaptivity in space and time, convergence results for one-step, multistep and peer-methods for the discretization of time-dependent PDE that can be interpreted as stiff ODEs in function spaces are summarized. In this case, most of the classical methods like Runge-Kutta-Rosenbrock and DG-methods suffer from order reduction, that is, the classical order cannot in general be achieved. I will discuss well established and also recently discovered opportunities to overcome this serious drawback. Throughout my talk I will present academic as well as real-life problems to illustrate the observations.

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Higher order space-time discretizations for hyperbolic and parabolic problems I: Motivation and schemes

Markus Bause[1] Uwe Köcher

In the field of numerical approximation of time dependent parabolic differential equations higher order variational time discretizations have recently attracted the interest of researchers. By using a Galerkin approach we have a uniform variational approach in space and time which is advantageous for the analysis of the fully discrete system as well for the construction of simultaneous space-time adaptive methods or residual-based stabilization schemes in the case of convection-dominated transport problems. Further, it is very natural to construct methods of higher order and well-known finite element concepts can be used to obtain at least A-stable discretizations.

In this contribution we present continuous and discontinuous Galerkin time discretizations for second order hyperbolic problems and a class of parabolic equations. Our interest in developing numerical approximation schemes of higher order accuracy for the hyperbolic wave equation comes from mechanical engineering. Material inspection by piezoelectric induced ultrasonic waves is a relatively new and an intelligent technique to monitor the health of light-weight structures (e.g. carbon fiber reinforced plastics), for damage detection (delamination, matrix cracks, fiber breaks) and non-destructive evaluation. Here, high accuracy of the numerical calculations is of particular importance. As far as parabolic problems are concerned, we are interested in studying coupled systems of convection-diffusion-reaction equations as they typically arise in mechanical and environmental engineering (e.g. flow and transport in the subsurface). Here, variational techniques might help in the future to develop new space-time stabilization techniques in the case of convection-dominated transport and to apply multiscale and upscaling techniques that are known from the spatial discretization to the time variable.

In this contribution we introduce the underlying mathematical models and present their space-time Galerkin discretizations. For the spatial discretization of the wave equation the interior penalty discontinuous Galerkin method is used. Mixed finite element approaches are applied to discretize the spatial variables of the parabolic problems. Qualitative properties of the discretization schemes are discussed.

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Higher order space-time discretizations for hyperbolic and parabolic problems II: Algebraic solver and numerical studies

Uwe Köcher[1] Markus Bause[2]

In this contribution we continue studying continuous and discontinuous Galerkin time discretizations for second order hyperbolic problems and parabolic equations. For the spatial discretization of the wave equation the interior penalty discontinuous Galerkin method is used. Mixed finite element approaches are applied to discretize the spatial variables of the parabolic problems.

First, starting from the discrete variational formulations we derive their algebraic counterparts. Then, we focus on the solution of the resulting block matrix systems. For the proposed space-time discretizations of the wave equation it is shown that the block matrix can be condensed algebraically such that the linear systems can be captured efficiently. For this condensation the block diagonal structure of the mass matrix is essential. For the mixed finite element approximation in space of the parabolic problems a non-hybrid variant of this discretization technique is applied. The solution of the indefinite system of equations is discussed. The implementation of the numerical schemes in a parallel finite element framework is addressed further.

Finally, the numerical performance properties of the schemes are carefully analyzed by means of numerical experiments. Convergence studies are presented and superconvergence properties of the temporal discretization are illustrated. In particular, for the approximation of the wave equation fourth order accuracy with respect to the discretization in time and space is demonstrated for the continuous approach in time.

References:

[1] U. Köcher, M. Bause: Variational space-time methods for the wave equation, Submitted.

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Stability of explicit time integration schemes for finite element approximations of linear parabolic equations with anisotropic meshes.

Lennard Kamenski[1] Weizhang Huang[2] Jens Lang[3]

We study the stability of the explicit time integration schemes for the linear finite element approximation of linear parabolic equations. A bound on the maximal possible time step is derived which is valid for any mesh and any symmetric positive definite diffusion matrix. Moreover, it is shown to be tight within a constant factor depending only on the space dimension. The geometric interpretation of the bound reveals that the stability condition depends of two factors. The first factor is proportional to the number of mesh elements and corresponds to the bound for a constant isotropic diffusion on a uniform mesh. The second factor reflects the effects of the mesh and shows that the alignment of the mesh with the major diffusion directions plays a crucial role in the stability condition. In particular, neither the eigenvalues of the diffusion matrix nor the mesh geometry on itself are important for the stability, but the matching between the mesh geometry and the inverse of the diffusion matrix. When the mesh is uniform in the metric induced by the inverse of the diffusion matrix, the stability condition is comparable to the situation with constant, isotropic diffusion problems on a uniform mesh. Numerical results are presented to verify the theoretical findings.

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Higher order Galerkin methods as time discretization for free surface flows

Stephan Weller[1]

Time discretization for free surface flows is a widely neglected problem. Explicit decoupling strategies lead to schemes that are only conditionally stable and existing semi-implicit schemes are of first order only. Some efforts in the direction of fully implicit and linearly implicit schemes have been made recently [1], but no stability or convergence results are available.

Galerkin methods in time have been successfully applied to a number of problems, i.e. parabolic PDEs [2], nonlinear ODEs [3] and also the Navier-Stokes equations [4]. An application to free surface flows is also available [5], it was however only used for a stability estimate and is of first order.

We present an application of a discontinuous in time Galerkin method to a free surface flow problem of arbitrary order. We use Arbitrary Lagrange Eulerian (ALE) coordinates on a problem-adapted mesh to deal with capillary or multiphase flows, allowing for a very precise representation of geometrical quantities.

Strategies to solve the resulting nonlinear coupled system of equations are discussed. Numerical results from a prototypical implementation are shown and some stability results are presented.

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[1] Bänsch, E. and Weller, S. *A comparison of several time discretization methods for free surface flows* in ALGORITMY, 19th Conference on Scientific Computing, Vysoke Tatry – Podbanske, Slovakia (eds Handlovicova, A., Minarechova, Z. & Sevcovic, D.)
[2] Thomée, V. *Galerkin Finite Element Methods for Parabolic Problems.* Springer Lecture notes in Mathematics (Springer)
[3] Schieweck, F. and Matthies, G. *Higher order variational time discretizations for nonlinear systems of ordinary differential equations.* Preprint, Otto-von-Guericke-Universität Magdeburg.
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[5] Bänsch, E. *Numerical methods for the instationary Navier–Stokes equations with a free capillary surface.* Habilitationsschrift, Universität Freiburg.

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Stability and efficiency of a recursive multirate Rosenbrock method

Karen Kuhn[1] Jens Lang[2]

Many physical phenomena contain different time scales. One way to solve the descriptive PDE is to discretize first in space and then apply a normal singlerate time integrator to the resulting ODE system. For problems with different time scales this might end up in very small time steps which have to be applied also to components with much less activity. That is why the application of multirate methods is reasonable. Different time step sizes are used for different components, depending on their individual activity. Since the stability character of a singlerate method usually is not carried over to the corresponding multirate method, we study the asymptotic stability for several multirate Rosenbrock methods. For several test problems we compare the needed CPU time of the multirate and the respective singlerate methods.

Acknowledgment: This work is supported by the "Excellence Initiative" of the German Federal and State Governments and the Graduate School of Computational Engineering at Technische Universität Darmstadt.

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[1] Savcenco, V. and Hundsdorfer, W.H. and Verwer, J.G., *A multirate time stepping strategy for stiff ODEs*, BIT, 47, 2007.
[2] Kuhn, K. and Lang, J., *Comparison of asymptotic stability for recursive ROS-multirate methods*, TU Darmstadt, Preprint 2625, 2011, submitted to the Journal of Comp. and Appl. Math. 2012.

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Finite Element simulations for a coupled bulk-surface convection diffusion problem

Kristin Held[1] Andreas Hahn[2] Lutz Tobiska[3]

We consider a convection diffusion equation in a bulk $\Omega \in \mathbb{R}^3$ coupled to a diffusion equation on its closed boundary Γ

$$\partial_t c - D\Delta c + (u \cdot \nabla)c = f \qquad \text{in}\,\Omega \tag{0.1}$$
$$D\partial_n c = -S(c, c_\Gamma) \qquad \text{on}\,\Gamma \tag{0.2}$$
$$\partial_t c_\Gamma - D_\Gamma \Delta_\Gamma c_\Gamma = g + S(c, c_\Gamma) \qquad \text{on}\,\Gamma. \tag{0.3}$$

The velocity field u is given and fulfills the pointwise constrains $\nabla \cdot u = 0$ and $u|_\Gamma = 0$. A linear and a nonlinear version of the source term are regarded

$$S(c, c_\Gamma) = k_a c - k_d c_\Gamma, \quad S(c, c_\Gamma) = k_a c(1 - c_\Gamma) - k_d c_\Gamma \quad (k_a, k_d > 0).$$

In this talk we present a higher order finite element discretization of the above problem on polyhedral domains. The bulk mesh consists of tetrahedrals and the surface mesh is its restriction to the surface and therefore triangular. For the basis functions on the surface we use a restriction of the bulk basis functions. Time discretization is made by a Crank-Nicolson scheme.

The equations are decoupled by an approximation of the source term based on former solutions for c and c_Γ, respectively. This leads to an iterative scheme in every timestep. In the n-th timestep for given c_n^k, $k \geq 0$, with $c_n^0 = c(t_{n-1})$ the surface equation (0.3) is solved as a diffusion reaction equation to get $c_{\Gamma,n}^k$. Using $c_{\Gamma,n}^k$ in equations (0.1) and (0.2) we get a Robin type convection diffusion problem. Its solution is a new approximation c_n^{k+1} of $c(t_n)$.

Using this decoupled algorithm we perform time dependent numerical simulations. We compare their steady states for $t \to \infty$ to the results of the associated stationary problems.

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Superconvergence for higher-order Galerkin FEM for convection-diffusion problems

Sebastian Franz[1] Hans-Görg Roos[2]

For singularly perturbed convection-diffusion problems and many numerical methods a *supercloseness property* is known for bilinear elements. This means that the difference between the Galerkin FEM solution u^N and the bilinear interpolant of the exact solution u is convergent of order two in the energy norm, although $u^N - u$ is only convergent of order one.

We will investigate similar properties for higher-order Galerkin FEM and look especially at the choice of suitable interpolation operators. Having a supercloseness property, it is cheap to obtain a better numerical solution by simple *postprocessing* — a superconvergent solution that enjoys a higher order convergence rate. We will address different possibilities for this kind of postprocessing.

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A priori error analysis of nonlinear singularly perturbed problems

Miloslav Vlasak[1] Vaclav Kucera[2]

We are concerned in the numerical analysis of the solution of semilinear singularly perturbed convection–diffusion equation. We assume discretization in space by discontinuous Galerkin method. In Kucera (2013), the method of lines as well as the implicit Euler method are analyzed and diffusion uniform error estimates are derived. We extend these results to midpoint rule, BDF-2 and time discontinuous Galerkin method of general order.

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On the role of the Helmholtz decomposition in mixed methods for incompressible flows and a new variational crime

Alexander Linke[1]

In incompressible flows with vanishing normal velocities at the boundary, irrotational forces in the momentum equations should be balanced completely by the pressure gradient. Unfortunately, nearly all available discretizations for incompressible flows violate this property. The origin of the problem is that discrete velocities are usually not divergence-free. Hence, the use of divergence-free velocity reconstructions is proposed wherever an L^2 scalar product appears in the discrete variational formulation, which actually means committing a variational crime. The approach is illustrated and applied to a nonconforming Crouzeix-Raviart finite element discretization. It will be proved and numerically demonstrated that a divergence-free velocity reconstruction based on the lowest-order Raviart-Thomas element increases the robustness and accuracy of an existing convergent discretization, when irrotational forces appear in the momentum equations.

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Adaptive refinement in isogeometric analysis

Bert Jüttler[1]

Numerical Simulation via Isogeometric Analysis is based on tensor-product NURBS (Non-Uniform Rational B-spline) parametrizations of planar and spatial computational domains. The NURBS technology has been developed in Computer Aided Geometric Design and is now the prevailing standard for representing free-form objects in CAD systems. However, this mathematical technology does not provide the possibility adaptive refinement, a feature which is strongly desired in numerical simulation. In order to overcome this limitation, several generalizations of tensor-product NURBS representations have been proposed. These include T-splines, hierarchical B-splines and locally refined B-splines. The talk will describe the various approaches and discuss their advantages and disadvantages. In particular we will focus on hierarchical B-splines since they seem to provide the best theoretical and practical properties.

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A geometry-independent framework for coupled volume and surface problems on evolving domains

Sebastian Westerheide[1] Christian Engwer[2]

Mathematical modeling of biological processes often results in a system of PDEs on a time-dependent domain $\Omega(t) \subset \mathbb{R}^{n+1}$, a system of PDEs on its n-dimensional surface $\Gamma(t) = \partial\Omega(t)$, and a certain coupling condition between both systems. In the course of this, $\Omega(t)$ and $\Gamma(t)$ frequently are of complex shape and given through image data, possibly with strong anisotropic deformations and changes in topology while evolving in time.

Classical numerical methods for simulations with this kind of models, such as Arbitrary Lagrangian-Eulerian schemes, use computational grids resolving the geometry. Thus, in case of anisotropic deformations or a changing topology, they require the handling of issues like remeshing. An efficient approach for such issues are numerical methods based on an implicit level set description of the domain and its surface.

We propose a numerical framework which uses level set based methods. In order to decouple the geometry of $\Omega(t)$ and the computational grid, we employ the unfitted discontinous Galerkin method [2,6] for the volume part of the problem. It takes benefit from properties and flexibility of discontinous Galerkin discretizations, e.g. local mass conservation. The surface part is treated by a newly developed, consistent extension of the unfitted discontinous Galerkin method which is inspired by [3,4]. A combination of both approaches enables the use of simple structured grids and removes the need to remesh in the considered case of time-dependent domains.

New infrastructure is described which integrates the framework in Dune [1] and Dune-PDELab and we show numerical experiments for a coupled model problem.

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Least squares methods with interface approximation for two phase Stokes flow

Fleurianne Bertrand[1]

We consider the coupled problem with Stokes flow in two subdomains separated by an interface. At the interface, continuity of the velocity and the momentum balance condition for the stress tensor need to be imposed. The interface is characterized by a level set function which satisfies an appropriate transport equation and the problem can be written as a domain decomposition problem.

In this talk we first present how the stationary Stokes problem can be written as a first order system. For numerical results a combination of H(div)-conforming Raviart-Thomas and standard H1-conforming elements is used.

After that we analyze the effect of approximated flux boundary conditions on Raviart-Thomas finite elements in order to get the effect of the approximated interface on the momentum balance condition. In particular, we present an estimate for the normal flux on interpolated boundaries.

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Effective boundary conditions for compressible flows over a rough surface

Giulia Deolmi[1] Wolfgang Dahmen[2] Siegfried Müller[3]

Domains with microscopic rough boundaries frequently arise in applications in engineering. For instance, space shuttles are often covered with tiles, while small air injecting nozzles are used over wings of aircrafts to reduce the drag [4]. Examples can also be found in nature, e.g. the skin of sharks [2], and in everyday life, e.g. golf balls.

Direct numerical simulations of a flow over a roughness are prohibitively expensive for small scale structures. If the interest is only on some macroscale quantity, it is sufficient to model the influence of the unresolved microscale effects. Such multiscale models rely on an appropriate upscaling strategy, the so called homogenization technique [1,3]. In this talk the strategy originally developed by Achdou et al. [1] for incompressible flows is extended to compressible high-Reynols number flows. For proof of concept a laminar flow over a flat plate with partially embedded roughness is simulated.

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Higher order variational time discretisations for convection-diffusion problems in time-dependent domains

Khalid Adrigal[1] Gunar Matthies[2]

We consider convection-diffusion equations in time-dependent domains where the movement of the domain boundary is prescribed. The time change of the domain is handled by the Arbitrary Lagrangian-Eulerian (ALE) formulation. It prevents strong mesh distortions which may occur for pure Lagrangian formulations since the given velocity of the domain boundary is extended to the mesh velocity inside domain in such a way that the mesh quality is preserved.

We will present conservative and non-conservative formulations of time-dependent convection-diffusion equations in time-dependent domains where special attention is paid to the time derivative and the mesh velocity.

To discretize in time, continuous Galerkin-Petrov methods (cGP) and discontinuous Galerkin methods (dG) as higher order variational time discretisation schemes are applied. These methods are known to be A-stable (cGP) or even strongly A-stable (dG).

The convergence properties of dG and cGP methods will be studied numerically. In the view to free boundary value problems, we will investigate how different approximations of the mesh velocity will influence the accuracy of the time discretisation schemes.

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Least-squares finite element methods for coupled generalized Newtonian Stokes-Darcy flow

Steffen Münzenmaier[1]

The coupled problem for an instationary generalized Newtonian Stokes flow in one domain and a generalized Newtonian Darcy flow in a porous medium is studied in this work. Both flows are treated as a first order system in a stress-velocity formulation for the Stokes problem and a volumetric flux-hydraulic potential formulation for the Darcy problem. The coupling along an interface is done by using the well known Beavers-Joseph-Saffman interface condition. A least-squares finite element method is used for the numerical approximation of the solution. It is shown that under some assumptions on the viscosity the least-squares functional corresponding to the nonlinear first order system is an efficient and reliable error estimator which allows for adaptive refinement of the triangulations. The adaptive refinement is examined in a numerical example where boundary singularities are present. Due to the nonlinearity of the problem a Gauss-Newton method is used to iteratively solve the problem leading to a sequence of well-posed variational problems. It is shown that the variational problems arising in the Gauss-Newton method are well-posed.

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Arbitrary order BEM-based FEM on star-shaped elements

Steffen Weißer[1]

In the development of numerical methods to solve boundary value problems the requirement of flexible mesh handling gains more and more importance. The BEM-based finite element method is one of the new promising strategies which yields conforming approximations on polygonal and polyhedral meshes, respectively. This flexibility is obtained by special trial functions which are defined implicitly as solutions of local boundary value problems related to the underlying differential equation. These functions are treated by means of boundary element methods (BEM) in the realization.

The first part of the presentation gives a short introduction into the BEM-based FEM and deals with recent developments. Here, the definitions of lower order trial functions are discussed for two and three space dimensions. Furthermore, it is shown that the method can be applied to mixed FEM formulations involving $H(\mathrm{div})$-conforming approximations on polygonal meshes. In the second part, ideas from the previous work [SIAM J. Numer. Anal., 50(5):2357–2378, 2012] are generalized to construct trial functions which yield arbitrary order of convergence. With the help of an appropriate interpolation operator it is possible to prove convergence rates in the H^1- as well as in the L_2-norm for the BEM-based FEM on polygonal meshes with star-shaped elements. Several numerical experiments confirm the theoretical results.

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Abstract L^2 analysis for variational inequalities of Signorini type

Linus Wunderlich[1] Barbara Wohlmuth[2] Olaf Steinbach[3]
Annalisa Buffa[4] Ericka Brivadis[5]

This talk is concerned with variationally consistent Lagrange multiplier based discretizations for Signorini type problems. We start with L^2 a priori estimates for well known low order finite element discretizations and then discuss modern discretizations within the framework of isogeometrical analysis.

While there is a series of results on the convergence rate in the H^1-norm for the primal solution and the $H^{-1/2}$-norm for the dual solution, there is hardly any result on the L^2-norm for the different solution components. Here we give new quasi-optimal results for low order discretizations based on the use of the Aubin–Nitsche trick. We point out that due to the inequality character of the problem the dual problem has reduced regularity, and thus standard techniques only yield sub-optimal results.

Motivated by these a priori results, we then focus on formulations based on the isogeometric approach. Here we present two alternatives: One is based on the use of a set of biorthogonal Lagrange multipliers, whereas the second one works with a trace space of possibly different order. Here we discuss advantages and disadvantages of these two strategies and provide abstract criteria for optimality.

Numerical examples illustrate the theoretical results and show quantitative and qualitative effects of the influence

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Topology optimization of electric machines based on topological sensitivity analysis

Peter Gangl[1] Ulrich Langer[2]

Topological sensitivities are a very useful tool for determining optimal designs. The topological derivative of a domain-dependent functional represents the sensitivity with respect to the insertion of an infinitesimally small hole. In the gradient-based ON/OFF method, proposed by N. Takahashi in 2005, sensitivities of the functional with respect to a local variation of the material coefficient are considered.

We show that, in the case of a linear state equation, these two kinds of sensitivities coincide. For the sensitivities computed in the ON/OFF method the generalization to the case of a nonlinear state equation is straightforward, whereas the computation of topological derivatives in the nonlinear case is ongoing work.

We will show numerical results obtained by applying the ON/OFF method in the nonlinear case to the optimization of an electric motor.

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Efficient numerical methods for the large-scale parallel solution of thermomechanical contact problems under consideration of mesh separation

Jörg Frohne[1] Heribert Blum[2] Andreas Rademacher[3] Korosh Taebi[4]

Our talk will treat two topics which will be joined in a model with the aim to simulate a deep hole drilling process. First we introduce a thermomechanical problem which describes elasto-plastic deformations with hardening coupled with a heat equation. After a brief introduction of the model and the variational equations we propose an integrated set of computational methods based on the finite element method for the large-scale parallel solution. Some results for three dimensional examples will demonstrate the efficiency of this approach. To discretize our problem we use piecewise linear or quadratic ansatz functions on a h-adaptive refined mesh.

In the second part we present an intuitive way to handle mesh separation e.g. caused by the cutting edge of the drill using a Discontinuous Galerkin (DG) method and reasonable physical criteria for the separation along an edge of an element. To enforce continuity between the DG elements as well as the Dirichlet boundary conditions we apply a Nitsche method. Of course this approach leads to a bigger amount of unknowns compared to a continuous approach but which can be reduced by using the DG elements only in a specific area around the separation zone. Since we treat the contact conditions with a primal dual active set method we have to think about if it is admissible to use DG elements along the contact boundary without losing the biorthogonality property. Finally we present some first results and an outlook.

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Non-linear FEM solvers on accelerator devices

Gundolf Haase[1] Manfred Liebmann[2] Caroline Mendonça Costa[3]

Solvers for linear system of equations have been accelerated by many-core hardware already in the past. As soon as discrete PDEs with time dependent or non-linear system matrices have to be investigated the acceleration of the overall code decreases rapidly. In these cases also matrix calculations and assembling have to be performed in the fast memory of the accelerator device.

We will present our all-in-one approach for non-linear elasticity problems in the framework of cardiovascular simulations. The achieved speedup in comparison to one CPU core ranges from 400 for the pure element matrix calculations to a factor of 10-30 for the overall non-linear solver. In case of a deforming geometry we present a mesh smoothing algorithm based on radial basis functions that is perfectly parallelizable on GPUs.

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SOFE - a new simple and sophisticated object oriented hp-FEM software in MATLAB

Lars Ludwig[1]

The majority of object oriented finite element software suffers from a high degree of complexity. Especially the large quantity of classes involved makes it hard to understand the role of individual components and their precise interplay among each other. However, without clearness and transparency for the applying researcher it becomes extremely difficult to implement new ideas, changes and amendments to the code that effect core concepts of the finite element method, such as setting up (hybrid)-discontinuous FE-spaces, realization of new interpolants, definition of a priori hp-meshes, computation of orders of convergence by means of reference solutions and many more.

We present a new, clearly structured and intuitive implementation of the hp-FEM in MATLAB that despite of its simplicity is able to tackle all kinds of problems and variations of the FEM in one, two and three space dimensions. After explaining the structure and flexibility of the code we demonstrate its features in various examples.

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Towards optimal performance of SeisSol, an unstructured ADER-DG code.

Alexander Nikolas Breuer[1] Alexander Heinecke[2] Sebastian Rettenberger[3]

SeisSol is one of the leading codes for earthquake scenarios, in particular for simulating dynamic rupture processes and for problems that require discretizing very complex geometries. Spatial adaptivity in 3D is realized by flexible unstructured tetrahedral meshes, using a high-order discontinuous Galerkin discretization and explicit time stepping following the Arbitrary high order derivatives approach. In the first part of the talk we show how hardware-aware programming of the computational kernels, dominated by small-rank matrix multiplication, in SeisSol leads to greatly improved performance on state-of-the-art supercomputing architectures. The second part discusses recent advances in preparing the clustered ADER time integration for large-scale simulations by taking requirements of hardware already in the numerical setup into account.

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First step towards parallel and adaptive computation of Maxwell's equations

Stefan Findeisen[1] Christian Wieners[2]

An electromagnetic wave consists of two fields, the electric $\mathbf{E}$ and the magnetic $\mathbf{H}$. They can be computed by the linear first–order Maxwell system

$$\mu\partial_t\mathbf{H} + \nabla\times\mathbf{E} = 0,\quad \varepsilon\partial_t\mathbf{E} - \nabla\times\mathbf{H} = 0,\quad \nabla\cdot(\mu\mathbf{H}) = 0,\quad \nabla\cdot(\varepsilon\mathbf{E}) = 0$$

with permeability μ and permittivity ε. Each field consists of three components. Hence one has to compute six components of the fields, which depend on space and time. For a given problem this can lead to huge linear systems. This is why fast space–time codes are needed to solve the problem in a reasonable computation time.

We use a discontinuous Galerkin finite element discretization with upwind flux for the spatial discretization. In order to avoid a CFL condition we use the implicit midpoint rule for time integration, which is of order two and allows larger time steps.

For discretization we define space–time cells $\tau := K_\tau \times I_\tau$ which consists of a spatial element K_τ and a local time interval I_τ. Hence the space–time domain $Q := \Omega \times (0,T)$ can be decomposed into a finite set $\mathcal{T}$ of open space–time elements $\tau \subset Q$ such that $\overline{Q} = \bigcup_{\tau\in\mathcal{T}} \overline{\tau}$. By using hash map containers to store the space–time cells, it is easy to distribute the space–time cells among the different processes and solve the problem in parallel.

Our code is p–adaptive and uses different time steps. In a first step we compute our discrete solution $\mathbf{u}_h$ on Q with fixed polynomial degree $p_\tau = 0$ for all $\tau \in \mathcal{T}$ and large step sizes. Then we take the flux over the space–time cell faces as an indicator where p_τ should be increased and space–time cells should be refined in time. Our code provides polynomials up to order four. After that we recompute the solution with the new distribution of the polynomial degrees and time refined space–time cells. Although taking the flux as a refinement indicator is heuristic, it leads to good results in a sense that high polynomial degrees and small time steps are used in areas where a single wavefront is located. In areas with absence of a wave, lowest polynomial degrees and larger time steps are used.

So far, we are able to solve the 2D reduction of Maxwell's equations. But in the future the described programming model will be the basis for a full 3D space–time adaptive code.

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A posteriori error estimates, stopping criteria, and adaptivity for multiphase compositional flows

Soleiman Yousef[1] Daniele Di Pietro[2] Martin Vohralik[3] Eric Flauraud[4]

In this paper we derive a posteriori error estimates for the compositional model of multiphase flow in porous media. We show how to control the dual norm of the residual augmented by a nonconformity evaluation term by fully computable estimators. We then decompose the estimators into the space, time, linearization, and algebraic error components. This allows to formulate criteria for stopping the iterative algebraic solver and the iterative linearization solver when the corresponding error components do not affect significantly the overall error. Moreover, the spatial and temporal error components can be balanced by time step and space mesh adaptation. Our analysis applies to a broad class of standard numerical methods, and is independent of the linearization and the iterative algebraic solvers employed. We exemplify it for the two-point finite volume method with fully implicit Euler time stepping, the Newton linearization, and the GMRes algebraic solver. Numerical results on two real-life reservoir engineering examples confirm that significant computational gains can be achieved thanks to our stopping criteria, already on fixed meshes.

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Adaptive anisotropic mesh refinement based hierarchical refinement indicators

Rene Schneider[1]

We propose a new refinement strategy, which based on hierarchical error estimates allows anisotropic mesh refinement of triangular elements. Combining element bisection, edge swapping and node removal operations, even re-alignment of the mesh with solution features of arbitrary direction is achieved and arbitrarily high aspect ratios can be automatically generated, starting from an isotropic coarse mesh. For problems with highly anisotropic solution features the discretisation error can be reduced by several orders of magnitude. Numerical experiments demonstrate the utility of the proposed anisotropic refinement strategy.

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The discrete compactness property for edge elements on anisotropic meshes

Ariel Lombardi[1]

In the theoretical analysis of finite element methods for Maxwell's equations we can distinguish two basic problems. The first one is to compute the eigenvalues (or resonant frequencies) of a bounded cavity. The second one is to compute the electromagnetic field in the cavity due to a known current source (at a nonresonant frequency). Edge finite elements have been used to approximate both problems, and the convergence was studied in several papers. The discrete compactness property is a useful tool for this analysis. It was first introduced by Kikuchi for edge elements of lowest order on tetrahedral shape-regular meshes.

The numerical approximation of both problems, and so, the validity of the discrete compactness property, have been considered in different situations by several authors: Boffi, Buffa, Caorsi, Costabel, Dauge, Fernandez, Hiptmair, Monk, Nicaise, Raffetto and others.

In this talk we extend some results of Nicaise and Buffa, Costabel and Dauge. Precisely, we present a proof of the discrete compactness property for tetrahedral edge elements of any order, on anisotropically refined meshes on a general Lipschitz polyhedral domain. We consider edge and corner refinements: our meshes are proposed in order to be able to adequately approximate the homogeneous Dirichlet problem for the Laplace operator with a right hand side in L^p for some $p \geq 2$.

The key ingredients of our approach are:

- Suitable decompositions of certain vector fields in $H_0(curl)$.
- Interpolation error estimates for edge elements of any order on anisotropic meshes satisfying the maximum angle condition.
- Control by below of the volume of the elements of the mesh in terms of the mesh-size parameter.
- Accurate error estimates for a continuous piecewise polynomial interpolation of the H^1_0-solution of the scalar Laplace equation with right hand side in L^p.

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Anisotropic mesh refinement in polyhedral domains: error estimates with data in $L^2(\Omega)$

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The presentation is concerned with the finite element solution of the Poisson equation with homogeneous Dirichlet boundary condition in a three-dimensional domain. Anisotropic, graded meshes [2] are used for dealing with the singular behavior of the solution in the vicinity of the non-smooth parts of the boundary. The discretization error is analyzed for the piecewise linear approximation in the $H^1(\Omega)$- and $L^2(\Omega)$-norms by using a new quasi-interpolation operator. This new interpolant is introduced in order to prove the estimates for $L^2(\Omega)$-data in the differential equation which is not possible for the standard nodal interpolant. These new estimates allow for the extension of certain error estimates for optimal control problems with elliptic partial differential equation and for a simpler proof of the discrete compactness property for edge elements of any order on this kind of finite element meshes, see [1].

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Breakfast: Buffet from 6:30 to 10:30

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Note: You have to choose the main course of the meal **each morning** (within the first session between 9:00 and 9:30).

The conference fee includes:

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- Tea, coffee, soft drinks and snacks during breaks.
- The conference dinner on Monday.

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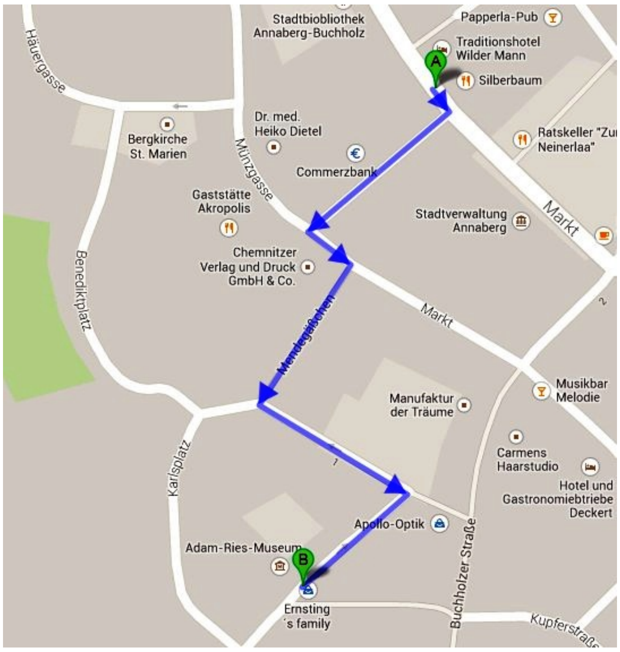

Only 250 m from the hotel you can find the Adam Ries Museum, a good advice if you are interested in history of mathematics. (www.adam-ries-museum.de)

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