Higher order space-time discretizations for hyperbolic and parabolic problems II: Algebraic solver and numerical studies

Uwe Köcher[1] Markus Bause[2]

In this contribution we continue studying continuous and discontinuous Galerkin time discretizations for second order hyperbolic problems and parabolic equations. For the spatial discretization of the wave equation the interior penalty discontinuous Galerkin method is used. Mixed finite element approaches are applied to discretize the spatial variables of the parabolic problems.

First, starting from the discrete variational formulations we derive their algebraic counterparts. Then, we focus on the solution of the resulting block matrix systems. For the proposed space-time discretizations of the wave equation it is shown that the block matrix can be condensed algebraically such that the linear systems can be captured efficiently. For this condensation the block diagonal structure of the mass matrix is essential. For the mixed finite element approximation in space of the parabolic problems a non-hybrid variant of this discretization technique is applied. The solution of the indefinite system of equations is discussed. The implementation of the numerical schemes in a parallel finite element framework is addressed further.

Finally, the numerical performance properties of the schemes are carefully analyzed by means of numerical experiments. Convergence studies are presented and superconvergence properties of the temporal discretization are illustrated. In particular, for the approximation of the wave equation fourth order accuracy with respect to the discretization in time and space is demonstrated for the continuous approach in time.

References:

[1] U. Köcher, M. Bause: Variational space-time methods for the wave equation, Submitted.

[1] Helmut-Schmidt-University Hamburg, Mechanical Engineering, Hamburg, Germany, koecher@hsu-hamburg.de

[2] bause@hsu-hamburg.de