Higher order Galerkin methods as time discretization for free surface flows

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Time discretization for free surface flows is a widely neglected problem. Explicit decoupling strategies lead to schemes that are only conditionally stable and existing semi-implicit schemes are of first order only. Some efforts in the direction of fully implicit and linearly implicit schemes have been made recently [1], but no stability or convergence results are available.

Galerkin methods in time have been successfully applied to a number of problems, i.e. parabolic PDEs [2], nonlinear ODEs [3] and also the Navier-Stokes equations [4]. An application to free surface flows is also available [5], it was however only used for a stability estimate and is of first order.

We present an application of a discontinuous in time Galerkin method to a free surface flow problem of arbitrary order. We use Arbitrary Lagrange Eulerian (ALE) coordinates on a problem-adapted mesh to deal with capillary or multiphase flows, allowing for a very precise representation of geometrical quantities.

Strategies to solve the resulting nonlinear coupled system of equations are discussed. Numerical results from a prototypical implementation are shown and some stability results are presented.

References:

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