First step towards parallel and adaptive computation of Maxwell's equations

Stefan Findeisen[1] Christian Wieners[2]

An electromagnetic wave consists of two fields, the electric $\mathbf{E}$ and the magnetic $\mathbf{H}$. They can be computed by the linear first–order Maxwell system

$$\mu\partial_t\mathbf{H} + \nabla\times\mathbf{E} = 0, \quad \varepsilon\partial_t\mathbf{E} - \nabla\times\mathbf{H} = 0, \quad \nabla\cdot(\mu\mathbf{H}) = 0, \quad \nabla\cdot(\varepsilon\mathbf{E}) = 0$$

with permeability μ and permittivity ε. Each field consists of three components. Hence one has to compute six components of the fields, which depend on space and time. For a given problem this can lead to huge linear systems. This is why fast space–time codes are needed to solve the problem in a reasonable computation time.

We use a discontinuous Galerkin finite element discretization with upwind flux for the spatial discretization. In order to avoid a CFL condition we use the implicit midpoint rule for time integration, which is of order two and allows larger time steps.

For discretization we define space–time cells $\tau := K_\tau \times I_\tau$ which consists of a spatial element K_τ and a local time interval I_τ. Hence the space–time domain $Q := \Omega \times (0,T)$ can be decomposed into a finite set $\mathcal{T}$ of open space–time elements $\tau \subset Q$ such that $\overline{Q} = \bigcup_{\tau\in\mathcal{T}} \overline{\tau}$. By using hash map containers to store the space–time cells, it is easy to distribute the space–time cells among the different processes and solve the problem in parallel.

Our code is p–adaptive and uses different time steps. In a first step we compute our discrete solution $\mathbf{u}_h$ on Q with fixed polynomial degree $p_\tau = 0$ for all $\tau \in \mathcal{T}$ and large step sizes. Then we take the flux over the space–time cell faces as an indicator where p_τ should be increased and space–time cells should be refined in time. Our code provides polynomials up to order four. After that we recompute the solution with the new distribution of the polynomial degrees and time refined space–time cells. Although taking the flux as a refinement indicator is heuristic, it leads to good results in a sense that high polynomial degrees and small time steps are used in areas where a single wavefront is located. In areas with absence of a wave, lowest polynomial degrees and larger time steps are used.

So far, we are able to solve the 2D reduction of Maxwell's equations. But in the future the described programming model will be the basis for a full 3D space–time adaptive code.

Acknowledgment: This work is supported by the DFG Research Training Group 1294 "Analysis, Simulation and Design of Nanotechnological Processes" at the KIT.

[1] Karlsruhe Institut of Technolgy, Departement of Mathematics, Kaiserstr. 12, 76128 Karlsruhe, Germany,
stefan.findeisen@kit.edu
[2] Karlsruhe Institut of Technolgy, Departement of Mathematics, Kaiserstr. 12, 76128 Karlsruhe, Germany,
christian.wieners@kit.edu