A geometry-independent framework for coupled volume and surface problems on evolving domains

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Mathematical modeling of biological processes often results in a system of PDEs on a time-dependent domain $\Omega(t) \subset \mathbb{R}^{n+1}$, a system of PDEs on its n-dimensional surface $\Gamma(t) = \partial\Omega(t)$, and a certain coupling condition between both systems. In the course of this, $\Omega(t)$ and $\Gamma(t)$ frequently are of complex shape and given through image data, possibly with strong anisotropic deformations and changes in topology while evolving in time.

Classical numerical methods for simulations with this kind of models, such as Arbitrary Lagrangian-Eulerian schemes, use computational grids resolving the geometry. Thus, in case of anisotropic deformations or a changing topology, they require the handling of issues like remeshing. An efficient approach for such issues are numerical methods based on an implicit level set description of the domain and its surface.

We propose a numerical framework which uses level set based methods. In order to decouple the geometry of $\Omega(t)$ and the computational grid, we employ the unfitted discontinous Galerkin method [2,6] for the volume part of the problem. It takes benefit from properties and flexibility of discontinous Galerkin discretizations, e.g. local mass conservation. The surface part is treated by a newly developed, consistent extension of the unfitted discontinous Galerkin method which is inspired by [3,4]. A combination of both approaches enables the use of simple structured grids and removes the need to remesh in the considered case of time-dependent domains.

New infrastructure is described which integrates the framework in Dune [1] and Dune-PDELab and we show numerical experiments for a coupled model problem.

References:

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