Novel Monte Carlo FE methods for random elliptic PDEs

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One of the key tasks in many areas of science and engineering is an efficient treatment of data uncertainties and the quantification of how these uncertainties propagate through the system. Subsurface flow is a prime example. The permeability of the subsurface is typically varying over many orders of magnitude, yet our information about the precise values is extremely limited. On the other hand subsurface flow simulations are paramount for safety assessments of longterm radioactive waste repositories or of novel hydrocarbon extraction techniques such as fracking. The mathematical challenges associated with uncertainty quantification are high-dimensional quadrature problems with integrands that involve the solution of PDEs with random coefficients. Due to the heterogeneity of the subsurface and the complexity of the flow, FE simulations of realisations of the integrand are very costly and so it is paramount to make existing uncertainty quantification tools more efficient. Although spectral methods, such as stochastic Galerkin or stochastic collocation type approaches, have recently been shown to be highly effective alternatives, due the curse of dimensionality their applicability is confined to $O(10 - 100)$ dimensions, which is not sufficient in subsurface flow. The only methods that do not suffer from this curse of dimensionality are Monte Carlo type methods. In this talk I will present new theoretical and numerical results on how to use deterministic Quasi-Monte Carlo sampling rules and hierarchies of finite element models (the so-called multilevel Monte Carlo method) to significantly accelerate the classical Monte Carlo method by a factor of 10-100 on realistic model problems. The analysis reduces to a careful application of classical regularity and finite element approximation error analysis.

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