Stability of explicit time integration schemes for finite element approximations of linear parabolic equations with anisotropic meshes.

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We study the stability of the explicit time integration schemes for the linear finite element approximation of linear parabolic equations. A bound on the maximal possible time step is derived which is valid for any mesh and any symmetric positive definite diffusion matrix. Moreover, it is shown to be tight within a constant factor depending only on the space dimension. The geometric interpretation of the bound reveals that the stability condition depends of two factors. The first factor is proportional to the number of mesh elements and corresponds to the bound for a constant isotropic diffusion on a uniform mesh. The second factor reflects the effects of the mesh and shows that the alignment of the mesh with the major diffusion directions plays a crucial role in the stability condition. In particular, neither the eigenvalues of the diffusion matrix nor the mesh geometry on itself are important for the stability, but the matching between the mesh geometry and the inverse of the diffusion matrix. When the mesh is uniform in the metric induced by the inverse of the diffusion matrix, the stability condition is comparable to the situation with constant, isotropic diffusion problems on a uniform mesh. Numerical results are presented to verify the theoretical findings.

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