A Petrov-Galerkin discretization with optimal test space of a mild-weak formulation of convection-diffusion equations in mixed form

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Inspired by the Discontinuous Petrov-Galerkin method [*Numer. Methods Partial Differential Equations*, 27 (2011), 70–105] by Demkowicz and Gopalakrishnan, we present a variational formulation of convection-diffusion equations, that is obtained by piecewise integrating one of the two equations in the system w.r.t. a partition of the domain into mesh cells. We apply a Petrov-Galerkin discretization with optimal test functions, or equivalently, minimize the residual in the natural norm associated to the variational form. These optimal test functions can be found by solving local problems.

The available freedom in the method is used to allow a (smooth) passing to a converging method in the convective limit, being a necessary condition to retain convergence and having a bound on the cost for a vanishing diffusion. With several numerical examples, the robust behavior of the method for vanishing diffusion will be illustrated.

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