The discrete compactness property for edge elements on anisotropic meshes

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In the theoretical analysis of finite element methods for Maxwell's equations we can distinguish two basic problems. The first one is to compute the eigenvalues (or resonant frequencies) of a bounded cavity. The second one is to compute the electromagnetic field in the cavity due to a known current source (at a nonresonant frequency). Edge finite elements have been used to approximate both problems, and the convergence was studied in several papers. The discrete compactness property is a useful tool for this analysis. It was first introduced by Kikuchi for edge elements of lowest order on tetrahedral shape-regular meshes.

The numerical approximation of both problems, and so, the validity of the discrete compactness property, have been considered in different situations by several authors: Boffi, Buffa, Caorsi, Costabel, Dauge, Fernandez, Hiptmair, Monk, Nicaise, Raffetto and others.

In this talk we extend some results of Nicaise and Buffa, Costabel and Dauge. Precisely, we present a proof of the discrete compactness property for tetrahedral edge elements of any order, on anisotropically refined meshes on a general Lipschitz polyhedral domain. We consider edge and corner refinements: our meshes are proposed in order to be able to adequately approximate the homogeneous Dirichlet problem for the Laplace operator with a right hand side in L^p for some $p \geq 2$.

The key ingredients of our approach are:

- Suitable decompositions of certain vector fields in $H_0(curl)$.
- Interpolation error estimates for edge elements of any order on anisotropic meshes satisfying the maximum angle condition.
- Control by below of the volume of the elements of the mesh in terms of the mesh-size parameter.
- Accurate error estimates for a continuous piecewise polynomial interpolation of the H_0^1-solution of the scalar Laplace equation with right hand side in L^p.

References:

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