

# On the role of the Helmholtz decomposition in mixed methods for incompressible flows and a new variational crime

Alexander Linke<sup>1</sup>

In incompressible flows with vanishing normal velocities at the boundary, irrotational forces in the momentum equations should be balanced completely by the pressure gradient. Unfortunately, nearly all available discretizations for incompressible flows violate this property. The origin of the problem is that discrete velocities are usually not divergence-free. Hence, the use of divergence-free velocity reconstructions is proposed wherever an  $L^2$  scalar product appears in the discrete variational formulation, which actually means committing a variational crime. The approach is illustrated and applied to a nonconforming Crouzeix-Raviart finite element discretization. It will be proved and numerically demonstrated that a divergence-free velocity reconstruction based on the lowest-order Raviart-Thomas element increases the robustness and accuracy of an existing convergent discretization, when irrotational forces appear in the momentum equations.

---

<sup>1</sup> Weierstrass Institute, Numerical Mathematics and Scientific Computing, Berlin, Germany,  
alexander.linke@wias-berlin.de