Fast domain decomposition solvers for discrete problems with chaotically subdomain wise variable orthotropism

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The second order elliptic equation is considered in the domain, composed of some shape and size irregular rectangles, which are nests of the orthogonal nonuniform decomposition mesh. The matrix of coefficients of the elliptic operator, written in the divergent form, is diagonal and its nonzero coefficients in each subdomain are arbitrary positive numbers. The orthogonal finite element mesh satisfies only one condition: it is uniform on each subdomain. No other conditions on the coefficients of the elliptic equation and on variable step sizes of the discretization and decomposition meshes are imposed. For the resulting discrete finite element problem, we present the DD (domain decomposition) preconditioner of the Dirichlet-Dirichlet type, in which d.o.f.'s at nodes of the decomposition mesh are split from others, and it is assumed that the contribution of the subproblem, related to these d.o.f.'s, to the computational cost of the DD solver is secondary. Essential components of DD preconditioner are the same as in the paper of Korneev, Poborchii & Salgado (2007). However, here we remove weak dependence of the relative condition number of the DD preconditioner on some measure of the local orthotropism of the discretization. We show that the DD solver has linear complexity, independently of the aspect ratios of the three types of orthotropism listed above. The result became possible due to the special way of the interface preconditioning by means of the inexact solver employing the preconditioner-multiplicator and the preconditioner-solver. The interface preconditioning makes also the main difference from other authors works on fast DD solvers for similar problems, *e.g.*, of Khoromskij & Wittum (1999,2004) and Kwak, Nepomnyaschikh & Pyo (2004).

Range of applications of the designed DD solver includes discretizations of a class of elliptic equations with non-matched deterioration of coefficients, a simple representative of which is $\mathcal{L}u \equiv y^2\partial^2 u/\partial x^2 + x^2\partial^2 u/\partial y^2$, $(x,y) \in \Omega = (0,1)\times(0,1)$, $u|_{\partial\Omega} = 0$. According to Korneev & Jensen (1997), the matrix of finite-difference analogue of $\mathcal{L}$ on the uniform square grid of size h can be also used for efficient preconditioning of the internal stiffness matrix of the reference p-element. In DD solvers for such problems, the number of subdomains, maximal over subdomains edge ratios and aspect ratios of orthotropism grow at $h \to \infty$. We show that nevertheless the linear complexity of DD solver is retained. At that the independent block of the DD preconditioner, related to the nodes of the decomposition mesh, can be preconditioned by the diagonal matrix.

Research is supported by the grant from the Russian Fund of Basic Research N 08-01-00676-a.
The author has been partially supported by the Johann Radon Institute for Computational and Applied Mathematics (RICAM) of the Austrian Academy of Sciences during his research visits at Linz.

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