Numerical simulation of elastic wave propagation in composite material

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Composites are one of the most promising materials to build light-weight structures for several fields of application, e.g. for wind energy plants and aerospace applications. Piezoelectric induced ultrasonic waves can be used for the development of structural health monitoring (SHM) systems. But, there are still a lot of open questions, especially regarding the application of ultrasonic waves in layered composite structures. Numerical simulation can help to understand wave propagation in heterogeneous and composite media. The reliable prediction of structural damages requires accurate numerical schemes.

Recently, there has been an increased interest in applying the discontinuous Galerkin method (DGM) to wave propagation. Some of the advantages of the finite element method are the flexibility with which it can accomodate the underlying geometry, discontinuities in the model and boundary conditions, and the ability to approximate the wavefield with high-degree polynomials. The DGM has the further advantage that it can accomodate discontinuities, not only in the media parameters, but also in the wavefield, it can be energy conservative, and it is suitable for parallel implementation.

In this contribution we study the application of continuous and discontinuous Galerkin methods for analyzing elastic wave propagation in heterogeneous media. A symmetric interior penalty discontinuous Galerkin method is used. An error analysis of this scheme is given in [1] for the accoustic wave equation. For the integration in time the second order-accurate method of Crank-Nicolson and the Newmark-Schemes are used. Precisely, we consider solving the linear-elastic wave equation without damping [2].

In our numerical results we study the performance and accuracy properties of continuous and discontinuous Galerkin methods for predicting wave propagation phenomena. In particular, a positive impact of higher order discontinuous approximations for heterogeneous materials will be illustrated; cf. also [3] for the application of higher order techniques to convection-dominated transport problems. Wave propagation in various composite materials and the potential of using guided waves for detecting structural damages is considered further.

The implementations were done by using the finite element toolbox `deal.II` [4].

References:

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