Reduced one-dimensional modelling and numerical simulation for mass transport in fluids

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Based on the Navier–Stokes equations in three space dimensions and a convection-diffusion equation, we use a nonlinear system of three hyperbolic PDEs in one space dimension to simulate mass transport. In this talk we focus on the modelling of mass transport through an arterial network. For the numerical treatment of the hyperbolic PDE system, we use stabilised discontinuous Galerkin (DG) approximations with a Taylor basis. Higher order discontinuous Galerkin approximations together with a suitable time integration method enable us to simulate wave propagtions for many periods avoiding excessive dispersion and dissipation effects. However higher order polynomials in standard discontinuous Galerkin approximations tend to create non-physical oscillations at sharp fronts and thus stabilisation techniques are required. Finally we present some numerical results illustrating the robustness of our model and the numerical discretisation.

References:

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