Crank-Nicolson discretization for parabolic optimal control problems with terminal observation

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Crank-Nicolson schemes are often used for the time discretization of parabolic partial differential equations. In this talk we present a second order discretization scheme for optimal control problems with terminal observation. The discretization is based on the Crank-Nicolson scheme with different time discretizations for state y and adjoint state p so that discretization and optimization commute.

As the scheme can be explained as the Störmer-Verlet method, we can also interpret the method in the context of geometric numerical integration of Hamilton systems as parabolic optimal control problems are Hamilton systems. This Hamiltonian structure is also discussed in this presentation. Further we point out that the scheme may also be obtained as a Galerkin method. We prove second order convergence of the scheme.

Finally we present the analytic solution of the optimal control problem as eigenfunction expansion with respect to the eigenfunction of the spatial operator. In a numerical example second order convergence in time is observed.

We discuss an optimal control problem with an inhomogenous intial condition for the state y. The resulting optimality system also contains an inhomogenous terminal condition for the adjoint state p.

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