Optimal control of coupled systems of ordinary and partial differential equations with algebraic constraints

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In the modelling of multi-body systems we find ODEs representing the interactions between the centers of mass, algebraic equations (AE) from constraining forces and elliptic PDEs modelling mechanical deformations within the bodies. Another typical example for coupled ODE, PDE and AE are free boundary problems, where, for example, the ODE describes the motion of the free interface between two phases and the PDE the evolution or quasi-stationary solution of quantities inside the phases. This might be coupled to constraints representing global conservation of mass. We discuss the well-posedness of these kind of problems shortly and are finally interested in the optimal control of these systems.

For solving the PDEs finite element methods are well applicable. Special attention has to be given to the finite elements in the neighbourhood of points or surfaces where forces are applied or whenever free boundaries occur. For several examples, e.g. an elastic crane, an elastic tyre-damper system, phase transitions and a fluid-elastomer interaction, the problems and first results are presented. The research on the elastic crane and elastic tyre is joint work with Matthias Gerdts and Roland Herzog.

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