Finite Element Method with local Trefftz basis functions on polygonal/polyhedral meshes

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In the development of numerical methods for boundary value problems, the requirement of flexible mesh handling gains more and more importance. D. Copeland, U. Langer and D. Pusch proposed a new kind of conforming finite element method on polygonal/polyhedral meshes in 2009. The idea is to use basis functions which are defined implicitly as local solutions of the underlying homogeneous problem with constant coefficients. The local problems are treated by means of boundary integral equations and are approximated by the use of the boundary element method in the numerics. Therefore, this promising strategy is also called BEM-based finite element method.

In the last years, there have been several developments concerning residual error estimates and extensions with higher order basis functions on polygonal meshes in two space dimensions. The challenging part is to handle polygonal elements and to get the constants in the approximation estimates independent of the polygonal shapes. Following recent ideas, error estimates can be proven which guarantee optimal rates of convergence in the H^1-norm as well as in the L_2-norm for a model problem. Concerning higher order approximations, the basic method has to be modified to work with a continuous approximation of the material coefficient.

After a short review of recent results, a generalization to three space dimensions is discussed. All theoretical results are confirmed and illustrated by several numerical experiments.

References:

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