Composite non-conforming elements and local projection stabilization for transport dominated flow problems

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We consider composite non-conforming elements which are suitable for solving both, convection-diffusion and Oseen equations, respectively. Numerical experiments show that the usual Local Projection Stabilization (LPS) fails in the case of non-conforming elements. This drawback can be cured by adding a modified LPS-term to the discrete bilinear form. In contrast to the known one-level LPS approach, the proposed discrete space of composite non-conforming elements does not need special enrichments. One advantage of the composite non-conforming elements compared to known conforming quadrilateral elements like the Taylor-Hood elements is that they guarantee a better local mass conservation. Another advantage is that the matrices for the coupling between pressure and velocity are much more sparse for the composite non-conforming elements. Furthermore, the low order non-conforming element (Crouzeix-Raviart element) leads to a diagonal mass matrix which is advantageous for time dependent problems. We present several numerical experiments which show optimal approximation and good stability properties for our proposed non-conforming elements and the modified LPS.

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