A nonlinear LPS FEM for convection-diffusion-reaction equations

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This contribution is devoted to the numerical solution of convection-diffusion-reaction equations by means of the finite element method (FEM). It is well known that the standard Galerkin discretization is inappropriate if convection dominates diffusion since the approximate solution is usually globally polluted by large spurious oscillations. The usual way of treating dominating convection consists in adding extra terms to the Galerkin formulation, aimed at enhancing the stability of the approximate solution. Among such stabilized FEMs, the local projection stabilization (LPS) method has received some attention over the last decade. Originally proposed for the Stokes problem and extended to the Oseen equations, the LPS method has also been used recently to treat convection-diffusion equations.

We introduce an extension of the LPS FEM and analyze it both in the steady-state and transient settings. In addition to the standard LPS method, a nonlinear crosswind diffusion term is introduced, which accounts for the reduction of spurious oscillations. The time-dependent equation is discretized in time with an implicit one-step θ-scheme. We prove the existence of a solution and, depending on the choice of the stabilization parameter, also its uniqueness. To our best knowledge, this is the first nonlinear discretization for convection-diffusion-reaction equations for which both existence and uniqueness of a solution can be shown. Moreover, we derive error estimates which are supported by numerical studies. These studies demonstrate also a reduction of the spurious oscillations.

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