Numerical studies of Galerkin-type time discretizations applied to transient convection-diffusion-reaction equations

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It is well known that the Galerkin finite element method is unstable for the numerical solution of convection-dominated problems since the solution is typically polluted by spurious oscillations. To enhance the stability while keeping the accuracy of the Galerkin method, several stabilization techniques have been developed. We consider the streamline upwind Petrov-Galerkin (SUPG) method and local projection stabilization (LPS) method to stabilize the Galerkin discretization. The SUPG formulation is strongly consistent whereas, the LPS method lies in the class of symmetric stabilizations and weakly consistent.

We shall employ the combination of SUPG and LPS methods in space with the variational type time-discretization schemes for the numerical solution of time-dependent convection-diffusion-reaction equations. In particular, we consider the discontinuous Galerkin (dG) and continuous Galerkin-Petrov (cGP) method to discretize the problem in time. Several numerical tests have been performed to assess the accuracy of the higher order time-discretization schemes. For the smooth solution, optimal order of convergence for cGP and dG-methods are obtained. Furthermore, numerical comparison of SUPG and LPS methods for the smooth solution shows that both stabilization techniques perform quite similar and no difference among them can be appreciated. Finally, the dependence of the results on the stabilization parameters are discussed for smooth and non-smooth solution.

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