Directional-do-nothing condition for the Navier-Stokes equations

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The numerical solution of flow problems usually requires bounded domains although the physical problem may take place in an unbounded or substantially larger domain. In this case, artificial boundaries are necessary. A well established artificial boundary condition for the Navier-Stokes equation discretized by finite elements is the "do-nothing" condition. The reason for this is the fact that this condition does appear automatically due to partial integration of the viscous term and the pressure gradient. This condition is one of the most established outflow conditions for Navier-Stokes but there are very few analytical insight into this boundary condition. We address the question of existence and stability of weak solutions for the Navier-Stokes equations with a "directional do-nothing" condition. In contrast to the usual "do-nothing" condition this boundary condition has enhanced stability properties. In the case of pure outflow, the condition is equivalent to the original one, whereas the new boundary condition has a dissipative effect in the case of inflow. We show existence of weak solutions and illustrate the effect of this boundary condition by computation of steady and for non-steady flows.

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