How a non-convergent Hessian recovery works in mesh adaptation

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A typical use of gradient/Hessian recovery in adaptive mesh computation is to compute an approximation to the gradient/Hessian of the exact solution based on the computed solution on the current mesh and then to generate a new mesh based on the recovered gradient/Hessian. Obviously, the convergence of such adaptive mesh computation relies crucially on the convergence of the recovery.

It is known that some gradient recovery methods are convergent on general meshes and even superconvergent for mildly structured meshes and for a type of adaptive meshes. The analysis of the Hessian recovery, on the other hand, receives much less attention and little is known except for special situations. Moreover, a convergent Hessian recovery cannot be obtained from the linear finite element approximation on general, non-uniform meshes. On the other hand, it has been observed that adaptive meshes generated by means of a non-convergent Hessian recovery still result in optimal order of error reduction.

In this talk we will try to explain why a non-convergent recovered Hessian works in mesh adaptation. We consider piecewise linear interpolation error and show that the interpolation error converges at a theoretically predicted order if the recovered Hessian satisfies some mild closeness conditions.

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