# Dirichlet boundary control problems: finite element discretization and error estimates

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This presentation is concerned with the unconstrained Dirichlet boundary control problem

$$\begin{aligned} \min J(y,u) = \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\alpha}{2}\|u\|^2_{L^2(\Gamma)}, \\ \text{s.t.} \quad -\Delta y = f \text{ in } \Omega, \\ y = u \text{ on } \Gamma, \end{aligned}$$

where the state equation has in general to be understood in the very weak sense. In order to solve this problem the state and adjoint state are discretized by linear finite elements and also the control is discretized by piecewise linear and continuous functions. For convex domains finite element error estimates of this problem are proven by May, Rannacher and Vexer (2009). However, numerical tests suggest that these results are not sharp. First theoretical results are presented, which fit to the observations. In addition, Dirichlet boundary control problems in non-convex domains are attacked. These are more sophisticated compared to the convex ones, since the optimal state does not belong to the space $H^1(\Omega)$. Thus, the weak formulation is not well defined. Instead one has to deal with the very weak formulation.

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