Non-conforming finite elements and Riccati-based feedback stabilization of the Stokes equations

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We investigate the problem of feedback stabilization of the Stokes equations. Following recent results by Raymond our goal is to apply a Riccati-based boundary feedback stabilization. Here the main difficulty is that a standard finite element discretization of this type of problems usually leads to a discrete differential-algebraic model of differential index two. It is, therefore, necessary to project the system to the space of divergence-free functions and apply the numerical methods for solving the Riccati equation to the resulting projected state space ODE system. However, it is prohibitive to form that system explicitly due to memory and complexity restrictions.

Here we consider non-conforming finite elements, which guarantee that an application of the spatially semi-discretized PDE operators always give a discrete divergence-free solution. So they do not provide the state space representation of the system, without algebraic constraints, but in the matrix based solver framework the projection of the solution never needs to be performed explicitly. As a second important ingredient of our solver, we show in a proof of concept implementation that the solver framework can in principle be implemented matrix-free, i.e., all steps of the underlying algorithms are expressed in terms of PDE operators, such that all applications of matrices are mapped to function calls within the finite element code.

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