Modelling and numerical experiments for Kirchhoff plates using finite strain

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In the simulation of deformations of plates it is well known that we have to use a special treatment of the thickness dependence. Therewith we achieve a reduction of dimension from $3D$ to $2D$.

For linear elasticity and small deformations several techniques are well established to handle the reduction of dimension and achieve acceptable numerical results. In the case of large deformations of plates with non-linear material behaviour there exist different problems. One of these is the impossibility of analytical integration over the thickness of the plate due to the non-linearities arising from the material law and the large deformations themselves. There are several approaches to introduce a hypothesis for the treatment of the plate thickness from the strong Kirchhoff assumption on one hand up to some hierarchical approaches on the other hand.

Here we consider a model of using the Kirchhoff assumption. Therewith, a fibre that is straight and perpenticular to the midsurface of the undeformed plate has to be straight and rectangular to the midsurface of the deformed plate, as well. A possible change in length of the fibre is not considered in this hypothesis. From now on, it is important that we avoid any further simplifications, which could be crucial for large strain.

We are aware of the fact, that the Kirchhoff assumption is well suited for small deformations and linear material laws. However, we want to investigate the deformation of thin plates with isotropic nonlinear material as a numerical experiment. Particularly we are interested in bending dominated real $3D$ - deformations. As a result we get a heavily deformed shell but without change in thickness.

This way of modelling leads to a two-dimensional strain tensor, which depends essentially on the first two fundamental forms of the deformed midsurface. By minimizing the resulting deformation energy we end up with a nonlinear equation, defining the unknown displacement vector $\mathbf{U}$. The aim of the presentation is to combine incremental Newton technique with the finite element discretisation. The first derivative of the energy functional is relatively easy to obtain (our nonlinear equation to solve), but its second derivative, for performing Newton's method, is analytically ambitious and leads to time consuming element routines. Nethertheless we demonstrate the practicability, due to the fast convergence of the Newton linearization.

We will present the total theory and give first numerical results for comparisions.

References:

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