Combination technique based k-th moment analysis of elliptic problems with random diffusion

Helmut Harbrecht[1] Michael Peters[2] Markus Siebenmorgen[3]

We consider the efficient deterministic solution of elliptic boundary value problems with random diffusion matrix. Assuming random perturbations with known k moments, we derive, to leading order in the random perturbation's amplitude, deterministic equations for k moments of the random solution. The solution's k-th moment satisfies a k-fold tensor product boundary value problem on the k-fold product domain which can efficiently be discretized in sparse tensor product spaces. By defining the complement spaces via Galerkin projections, the related system of linear equations decouples and can be solved by standard multilevel finite element solvers. Numerical results for $k = 2$ are presented to validate and quantify our theoretical findings.

[1] Universität Basel, Departement Mathematik und Informatik, Basel, Schweiz, helmut.harbrecht@unibas.ch

[2] Universität Basel, Departement Mathematik und Informatik, Basel, Schweiz, michael.peters@unibas.ch

[3] Universität Basel, Departement Mathematik und Informatik, Basel, Schweiz, markus.siebenmorgen@unibas.ch