Two-level domain decomposition preconditioners for multi-phase elastic composites

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We analyse two-level overlapping Schwarz domain decomposition methods for a finite element discretization of the PDE system of linear elasticity. The focus in our study lies in the application to particle-reinforced composites with nearly incompressible and very stiff inclusions with large jumps in their material coefficients. We present explicit bounds for the condition number of the two-level additive Schwarz preconditioned linear system. Thereby, we do not require that the coefficients are resolved by the coarse mesh. The bounds show a dependence of the condition number on the energy of the coarse basis functions, the coarse mesh, and the width of the overlap. Similar estimates have been developed for scalar elliptic PDEs by Graham, Lechner and Scheichl (2007). The coarse spaces of our method are assumed to contain the six rigid body modes and can be considered as generalizations of the space of piecewise linear vector valued functions on a coarse triangulation. The developed estimates provide a concept for the construction of coarse spaces which can lead to preconditioners which are robust w.r.t. discontinuities in the elasticity coefficients of the underlying composite.
In our numerical tests, we extend the linear multiscale finite element method as formulated by Hou and Wu (1997) to the system of linear elasticity. For isolated inclusions of high contrast in the interior of coarse elements, the condition number bound does not depend on variations in the Young's modulus and the Poisson's ratio for the multiscale coarse space. By using oscillatory boundary conditions of the multiscale basis functions, the method is robust also in cases where inclusions cross coarse element boundaries. Furthermore, linear and energy minimizing coarse spaces are discussed.

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