On adaptive FEM for viscoelasticity at large strain deformations

Hansjörg Schmidt[1] Arnd Meyer[2]

Following the derivation in [1] we arrive at the following (non-linear) weak formulation for large deformations

$$0 = \int_{\Omega_0} \overset{2}{T} : E\left(U(t);V\right)\ \mathrm{d}\Omega_0 - \int_{\Omega_0} \rho_0 \vec{f}(t)\cdot V\ \mathrm{d}\Omega_0 - \int_{\Gamma_N} \vec{g}(t)\cdot V\ \mathrm{d}\mathcal{S}_0 \quad \forall V \in \mathbb{V},\ \forall t \in [0, t_{end}]\,.$$

The dependence of the second Piola-Kirchhoff stress tensor $\overset{2}{T}$ on the displacement U and potential other quantities has to be specified. Polymers and soft tissues are often modelled as viscoelastic continua which are characterised by a spontaneously elastic and dissipative viscous behaviour. There are at least two different models for viscoelasticity. In [2, p. 242] the model of fading memory is used to arrive at the stress–strain relation

$$\overset{2}{T}(t,U) = \mathfrak{h}_e\left(C(U(t))\right) + \int\limits_0^\infty \mathfrak{G}(C(U(t));s) : (\Delta C(t-s,t) - I)\ \mathrm{d}s$$

with a function $\mathfrak{G}(C;\cdot)$ which is pointwise a fourth order material tensor and where the right Cauchy-Green tensor $C(U(t))$ depends solely on the current displacement, but $\Delta C(t-s,t)$ depends on the whole history of the displacement U. This stress-strain relation is accurate for large strain deformations which are slow with respect to the properties of the considered material. Another model for the stress–strain relation is based on thermodynamic with internal state variables, [3], and results in

$$\overset{2}{T}(t,U) = 2\left.\frac{\partial\psi(C,G)}{\partial C}\right|_{C(U(t)),G(t)} \quad \text{subject to} \quad \mathcal{F}\left(t,G,\dot{G},C\right) = 0,$$

with a differential(-algebraic) equation describing the evolution of the internal variable G.

The aim is to improve the simulation of these materials, to achieve results with a higher accuracy with lower computational cost. We give some remarks about the discretisation and adaptivity in time and space.

References:

[1] Meyer, A., "Error estimators and the adaptive finite element method on large strain deformation problems", Mathematical Methods in the Applied Sciences, vol. 32, no. 16, John Wiley & Sons, Ltd., pp. 2148–2159, 2009

[2] Coleman, B.D. and Noll, W., "Foundations of Linear Viscoelasticity", Rev. Mod. Phys., Vol. 33, No. 2. (1961), pp. 239-249.

[3] Coleman, B.D. and Gurtin, M.E., "Thermodynamic with internal state variables"(1967), http://repository.cmu.edu/math/83

[1] Chemnitz University of Technology, Dep. of Math., Reichenhainer Straße 41, 09126 Chemnitz, hanss@hrz.tu-chemnitz.de

[2] Chemnitz University of Technology, Dep. of Math., a.meyer@mathematik.tu-chemnitz.de