A two-level local projection stabilisation on uniformly refined triangular meshes

Gunar Matthies[1] Lutz Tobiska[2]

The local projection stabilisation (LPS) has been successfully applied to scalar convection-diffusion-reaction equations, the Stokes problem, and the Oseen problem.

A fundamental tool in its analysis is that the interpolation error of the approximation space is orthogonal to the discontinuous projection space. It has been shown that a local inf-sup condition between approximation space and projection space is sufficient to construct modifications of standard interpolations which satisfy this additional orthogonality.

There are different versions of the local projection stabilisation on the market; we will consider the two-level approach based on standard finite element spaces Y_h on a mesh $\mathcal{T}_h$ and on projection spaces D_h living on a macro mesh $\mathcal{M}_h$. Hereby, the finer mesh is generated from the macro mesh by a certain refinement rules. In the usual two-level local projection stabilisation on triangular meshes, each macro triangle $M \in \mathcal{M}_h$ is divided by connecting its barycentre with its vertices. Three triangles $T \in \mathcal{T}_h$ are obtained. Then, the pairs $(P_{r,h}, P^{\text{disc}}_{r-1,2h})$, $r \geq 1$, of spaces of continuous, piecewise polynomials of degree r on $\mathcal{T}_h$ and discontinuous, piecewise polynomials of degree $r-1$ on $\mathcal{M}_h$ satisfy the local inf-sup condition and can be used within the LPS framework.

One disadvantage of this refinement technique is however that $\mathcal{T}_h$ contains simplices with large inner angles even in the case of a uniform decomposition $\mathcal{M}_h$ into isosceles triangles. Another drawback is that this refinement rule leads to non-nested meshes and spaces whereas the common refinement technique of one triangle into 4 similar triangles (called red refinement in adaptive finite elements) results into nested meshes and spaces.

We will show that in the two-dimensional case the pairs $(P_{r,h}, P^{\text{disc}}_{r-1,2h})$, $r \geq 2$, satisfy the local inf-sup condition with the refinement of one triangle into 4 triangles. Consequently, the LPS can be also applied on sequences of nested meshes and spaces and keeping the same error estimates. Finally, we compare the properties of the two resulting LPS methods based on the different refinement strategies by means of numerical test examples for convection-diffusion problems with dominating convection.

References:

[1] G. Matthies, L. Tobiska: A two-level local projection stabilisation on uniformly refined triangular meshes. accepted for Numer. Algorithms 2012. DOI: 10.1007/s11075-012-9543-4.

[1] Universität Kassel, FB 10, Institut für Mathematik, Kassel, Germany,
matthies@mathematik.uni-kassel.de

[2] Otto-von-Guericke-Universtität Magdeburg, Institut für Analysis und Numerik, Postfach 4120, 39016 Magdeburg, Germany,
tobiska@ovgu.de