A high order discontinuous Galerkin method for the Boltzmann Equation

Gerhard Kitzler[1] Joachim Schöberl[2]

The Boltzmann equation is a statistical model for gases. The density distribution function $f(t, x, v)$ describes the propability to find a particle at time t near the spatial position x and which has the velocity close to v. The time evolution of f is given by the Boltzmann equation. The collision of particles is formulated in terms of the collision operator $Q(f)$ which is local in x and t. We perform a Petrov-Galerkin method in the spatial domain Ω and velocity domain $\mathbb{R}^3$. In the v domain the solution is expanded as a sum over multivariate Lagrange polynomials $l_j(x)$ times an appropriate gaussian peak. In space we discretize by a high order discontinuous Galerkin method with natural upwind fluxes.

Due to this expansion, the Boltzmann transport operator decouples when using Gauß-Hermite integration rules of appropriate order into transport operators for the individual components.

[1] TU Wien, Institute for Analysis and Scientific Computing, Wiedner Hauptstraße 8, 1040 Wien, gerhard.kitzler@tuwien.ac.at

[2] TU Wien, Institute for Analysis and Scientific Computing, Wiedner Hauptstraße 8, 1040 Wien, joachim.schoeberl@tuwien.ac.at