Analysis of preconditioned iterative methods for the Helmholtz equation

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The finite element discretization of time-harmonic wave propagation problems, such as the Helmholtz equation or the Maxwell equations, leads a to solution of large, complex valued, indefinite, non-hermitian and non-normal linear systems. These systems are typically solved with a suitable preconditioned iterative method. Due to indefiniteness and non-normality of the linear system, a rigorous convergence analysis of such iterative solution methods is still a challenge.

Because of the non-normality, the eigenvalues alone do not give information of the convergence. In addition, due to indefinteness, the interesting eigenvalue is the one closest to the origin. This eigenvalue cannot be bounded using techniques familiar from convergence analysis of elliptic problems. In this talk we discuss these difficulties and possible solutions to them. As a model problem, we use the Helmholtz equation discretized with the standard first order finite element method, and solved using the preconditioned GMRES method.

The non-normality can be handeled in analysis by using a suitable convergence criterion. For GMRES, two possiblities exist , estimating the location of the field of values or the pseudospectrum. The FOV based convergence criterion has been used to study Laplace preconditioners for Helmholtz equation with losses. The major shortcomings of FOV based convergence criterion are in handling the indefiniteness of the linear system. These problems with FOV based criterion motivate us to study a pseudospectrum based criterion. The pseudospectrum is well suited for handling indefinite problems. We demonstrate this for the Helmholtz equation with absorbing bounary conditions.

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