

# Towards the direct and inverse adaptive mixed finite element formulations for nearly incompressible elasticity at large strains

Anke Bucher<sup>1</sup> Uwe-Jens Görke<sup>2</sup> Reiner Kreißig<sup>3</sup>

This contribution presents advanced numerical models for the solution of the direct and inverse problems of nearly incompressible hyperelastic processes at large strains. The discussed mixed finite element approach contributes to the numerical simulation of coupled multiphysics problems, including the calibration of appropriate material models (parameter identification). The presented constitutive approach is based on the multiplicative decomposition of the deformation gradient resulting in a two-field formulation with displacement components and hydrostatic pressure as primary variables. The ill-posed inverse problem of parameter identification analyzing inhomogeneous displacement fields is solved using deterministic trust-region optimization techniques. Within this context, a semi-analytical approach for sensitivity analysis represents an efficient and accurate method to determine the gradient of the objective function. The mixed boundary value problem is based on the spatial discretization of the weak formulations of the linear momentum balance and the incompressibility condition. Its linearization serves as basis for the solution of the direct problem, while the implicit differentiation of the weak formulations with respect to material parameters provides the necessary relations for the semi-analytical sensitivity analysis. Adaptive mesh refinement and mesh coarsening are realized controlled by a residual a posteriori error estimator. Efficiency and accuracy of the presented direct and inverse numerical techniques are demonstrated on a typical example.

---

<sup>1</sup> Leipzig University of Applied Sciences (HTWK), Mechanical and Energy Engineering,  
bucher@me.htwk-leipzig.de

<sup>2</sup> Helmholtz Centre for Environmental Research – UFZ, Department of Environmental Informatics ,  
uwe-jens.goerke@ufz.de

<sup>3</sup> Chemnitz University of Technology, Institute of Mechanics,  
reiner.kreissig@mb.tu-chemnitz.de