Numerical aspects of plates under large deformations

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Elastic material behavior of plates, more precisely thin and plane mechanical structures, often leads to large deformations. Still today, the elastic material models are mainly phenomenological, that is suggested formulas for the stored elastic energy Ψ are verified by experiments. Instead of starting from the balance of forces, we base our model on a stored energy minimisation $\Psi \to \min$ which leads to an equivalent variational equation as the balance of forces. The advantage of this approach lies in the use of optimisation strategies like line-search methods to improve the convergence of the newton-solver. To simulate very thin structures like plates, one approach is a reduction to the two-dimensional midsurface by restricting the deformation in a certain way. We use a polynomial ansatz over the thickness η^3, e.g.

$$x(\eta^1,\eta^2,\eta^3) = X(\eta^1,\eta^2,\eta^3) + U(\eta^1,\eta^2) + \eta^3\theta_1(\eta^1,\eta^2) + (\eta^3)^2\theta_2(\eta^1,\eta^2) + (\eta^3)^3\theta_3(\eta^1,\eta^2).$$

We insert this special deformation in our three-dimensional model and get a non-linear two-dimensional PDE. This reduction allows more complex deformations than the Midlin-Reisner or Kirchhoff-model, especially change in thickness and shear. We present numerical experiments (implemented in FEniCS) and discuss the how the polynomial degree of the ansatz effects the solution and how the integration over the thickness is realizied.

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